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# Three essays on strategic trade policy: precommitment, time consistency, and effects of a ratio quota

Chongkook Park  
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**Three essays on strategic trade policy: Precommitment, time consistency, and effects of a ratio quota**

**Park, Chongkook, Ph.D.**

**Iowa State University, 1990**

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**Three essays on strategic trade policy: Precommitment, time  
consistency, and effects of a ratio quota**

by

Chongkook Park

A Dissertation Submitted to the  
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Ames, Iowa  
1990

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## 1. INTRODUCTION

### 1.1 Overview

The theory of strategic trade policy is now well established as a branch in international economics. Many agree that the assumptions of perfect competition are not valid for a large number of markets in which significant international trade occurs. In many markets, firms compete against each other in one market to sell (close) substitutes, or complements, where each firm has some market power. Thus, it can be argued that international competition is in many cases, oligopolistic, because products are differentiated, or because the number of firms is small due to entry barriers or due to government actions which result in implicit cartellization of its own firms in those markets.

One reason that a firm seeks market power is that a firm under oligopolistic competition may earn a positive economic profit. Governments may wish to help domestic firms capture large shares of profit-earning industries for both economic and political reasons. Various policy variables may be employed, including subsidies and tariffs, to support domestic firms. As in the domestic distortions literature, and the optimal tariff literature, the presence of imperfectly competitive markets, with the ensuing market failure and the implied national market power, provides an incentive for government intervention. However, the focus of the strategic trade

policy literature is different since it focuses on the effect of government policies on the strategic interaction between domestic and foreign firms.

The strategic trade theory is, in fact, a theory that resulted from a “new thinking” about trade policy, and this new thinking has grown with three most important elements according to Krugman (1986). First, the importance of world trade to the U.S. has been steadily increasing. In 1960, U.S. firms considered the domestic market to be more significant than the world market. They now face important foreign rivals in the U.S. market, or tough rivals in foreign countries. Secondly, trade occurs, if for mutual benefits, not only because there are differences in factor endowments between countries, but also because countries have and fully utilize different technologies. Thus, “trade seems to reflect arbitrary or temporary advantage resulting from economies of scale, or shifting leads in close technological races” (Krugman, 1986, p. 7). The dominance of Japanese firms in the world market of electronics very well reflects this view. Finally, thanks to major innovations in the field of industrial organization in 1970s and 1980s, the strategic behavior of rivalry among firms is studied in new international economics. When a firm invests in excess capacity that it does not intend to use, the presence of this excess capacity deters potential entrants and allows the firm to enjoy positive economic (or super normal) profits. Thus, one implication of this new theory is that governments in many countries may actively pursue profit-shifting policies toward its firms.

However, it is important to recognize that the theory of strategic trade policy, with its conclusion that increasing returns to scale are also a cause of trade along with comparative advantage, and that many international markets are imperfectly

competitive, has drawn many criticisms.<sup>1</sup> First, the new theory assumes that the active government has access to the same (complete) information set as the firms in the industry under the examination and that it acts in a way that reflects the national interest, both of which are highly questionable assumptions.

Secondly, the theory has empirical difficulties in formulating the effective strategic policies. For the strategic policy to be effective, we should know detailed information on cost and demand structures of the foreign firms as well as, of home firms. We also should have reliable models of how oligopolists behave; they are cooperative or noncooperative; they are price-setter or quantity-setter. Furthermore, the empirical studies become more difficult when firms in question play a multi-stage game whose rules and objectives are complex and obscure.

Thirdly, the theory inherently does not consider a general equilibrium aspect. To promote particular sectors means that a government must draw resources away from the other sectors. A would-be loss from the other sectors which are not supported may dominate the gain from the targeted industry.

Fourth, the strategic trade policy ignores retaliation by the foreign government. Once retaliation is induced, free trade emerges.

Lastly, and most importantly, the new trade theory ignores the political economy of domestic politics. The special interest groups are omnipresent (and to a degree, omnipotent). The strategic trade policy may raise welfare of small, fortunate groups by large amount at costs of larger, more diffuse groups.

In this dissertation, we correct some errors which appeared in earlier papers in strategic trade policy, supplement some important notions such as "time consis-

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<sup>1</sup>See Krugman (1987).

tent” solutions, and investigate the effects of a proportional quota under Cournot equilibrium.

We introduce papers in this area in the next section, and provide the objectives of the dissertation at the end of this chapter.

## 1.2 Literature Review

Papers on strategic trade policy may be categorized as focusing on the following issues: i) increasing returns to scale as a cause of trade, ii) profit-shifting motive as a focus for policy intervention, and iii) extraction of rents from a foreign monopolist. To study the ‘time inconsistency’ problem in a paper utilizing a multi-stage game, we also present papers on ‘time consistency’ in trade model. Then, papers which investigate the effects of the VER are also surveyed.

Before we begin, it is important to recognize that the results of strategic policy depend on the relationship between goods produced by the competing firms. Thus, strategic substitutes and complements are defined by Bulow, Geanakoplos and Klemperer (1985). Goods are strategic substitutes (complements) if a more aggressive strategy (lower price or higher quantity) by player one lowers (raises) player two’s marginal profits. This definition will be applied whenever it is appropriate.

### 1.2.1 Increasing Returns: A Cause of Trade

Krugman (1979) showed that trade (and hence gains from trade) will occur due to increasing returns to scale, which are internal to firms, even when there are no differences in tastes, technology, or factor endowments. In particular, firms in oligopolistic competition take trade as a way of extending the market and allowing



exploitation of scale economies, which may be more appropriate in explaining the current (and future) international trade than perfect competition is.

Krugman (1980) investigates the effect of transportation costs to show that countries with larger domestic markets will, other things equal, have higher wage rates and will tend to export those goods for which they have relatively large domestic markets. He employs almost the same model as Krugman (1979).

Rauch (1989) shows that country size and the degree of increasing returns displayed by any goods may not be the sole cause of trade. Necessary and sufficient condition for a country to be a net exporter in a particular good requires geographic concentration in producing the good. In Sweden, Volvo and Saab are located at two different cities. Thus, even the city size is small relative to the population of a country, efficiency of city formation rather than country size dictates comparative advantage with respect to IRS goods.

Markusen (1981) shows how imperfect competition can form a basis for trade, and whether there are gains (or losses) from trade under this assumption. He employs a two goods, two country model such that the production of one good was monopolized in each country. Under the assumption of Cournot–Nash behavior, it is shown that trade will lead to a bilateral welfare improvement when countries are identical in all respects. Alternate stable equilibria in which only one country produces the monopolized goods necessarily involve an increase in total world real income relative to autarky, but do not necessarily involve gains for both countries.

### 1.2.2 Profit-Shifting Motive

Spencer and Brander (1983) have pioneered this area to show, using two-, or three-stage game-theoretic models, that if the government can act first, then its action will change final outcomes from those played by firms. Assuming a structure in which research and development (R & D) decisions are made before production decisions, and in which two firms, one foreign and one domestic, act as Cournot competitors at each stage of the game, they analyze how a country can use subsidies on R & D and export to improve domestic welfare. They show that if either investment or export subsidies are available, it is optimal to choose a positive level for that policy. However, they also show that when both policy instruments are available, the active government(s) should tax investment and subsidize the export. They claim to show that the investment tax, if chosen optimally by the government, achieves cost minimization (or efficiency).

Brander and Spencer (1984) extend the basic argument for intervention to situations in which duopolistic competition takes place in the importing (home) market. They show that an import tariff is often beneficial in these cases.

Brander and Spencer (1985), which is a companion paper to Spencer and Brander (1983), develop a model in which one home firm and one foreign firm produce perfectly substitutable goods and compete in a third country. They consider a Cournot-Nash equilibrium, and find that if the home country's government can credibly precommit itself to pursue a particular trade policy before firms make production decisions, then an export subsidy is optimal. When two governments pursue a subsidy policy, then the noncooperative Nash subsidy equilibrium is characterized by positive production subsidies in both exporting countries. When the importing country has an incentive

to set a tariff or subsidy on the import, then the optimal tariff is positive if the rate of change of the consumer price with respect to the tariff is less than one. This latter result is similar to Brander and Spencer (1984).

Eaton and Grossman (1986) analyze trade and industrial policy under oligopoly, using a conjectural variations approach. There are two firms, one home and one foreign, whose products may be perfect or imperfect substitutes. Two firms compete in the home country. In the duopoly case, profits can be shifted only if the firms' conjectural variations differ from the true equilibrium responses that would result if they were to alter their output levels. The choice between a tax and a subsidy in this case depends on whether the home firm's output in free trade equilibrium exceeds or falls short of the level that would emerge under "consistent"<sup>2</sup> or Stackelberg conjectures. When there is more than one domestic firm, competition among them is detrimental to home country social welfare. A production or export tax will lead domestic firms to restrict their outputs, shifting them closer to the level that would result with collusion. The tax, thus, enables the home government to exploit its monopoly power in trade fully.

Carmichael (1987) considers a two-stage Bertrand duopoly game to demonstrate how the sequence of firm and government actions can explain the origins of export credit subsidies, and can affect the welfare consequences of international agreements to adopt subsidy rate ceilings. Under competitive subsidies, the firms act first by

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<sup>2</sup>The notion of 'consistent conjectural variations' was introduced in economics by Bresnahan(1981). Then, many in trade theory have applied that concept. But, it was short-lived both because Boyer and Moreaux (1983) showed that any sustainable output combination can be obtained as a locally consistent conjectures equilibrium, and because game theorists argue that conjectures are not meaningful in a static game. See Cheong(1988).

stating prices. If at least one firm can inflate the stated price successfully, all rents go from the government to the firms. Hence, there is no change in welfare. Under controlled subsidies, the government acts first by adopting subsidy rate ceilings that bind. If the government neglects the effects of the ceiling upon the stated prices, they would adopt ceilings to raise welfare. If the governments adopt ceilings greater than zero, they would reduce ceilings. During an iterative process of ceiling and price adjustments that leads to a new equilibrium, welfare is lower.

Gruenspecht (1988) investigates the effects of export subsidies for differentiated products. He assumes two firms (one home and one foreign) behave as Bertrand players with differentiated goods in a third country. Firms set price before governments set subsidy levels. Positive subsidies can be optimal. Adoption of a subsidy program by one country does not necessarily hurt the unsubsidized rival. Subsidies are rent shifting, but also facilitate collusion.

### **1.2.3 Extraction of Rents**

Brander and Spencer (1981) consider a tariff as a tool to extract a rent from a foreign monopolist. Since an international monopolist earns a positive rent, a country importing such a good pays rent. In the absence of potential (Stackelberg) entry, a tariff extracts some of rent not from domestic residents, but from foreigners. Under a potential entry, the foreign monopolist will change its policy and the welfare of the domestic country will be improved. The Stackelberg leader-follower model can lead to intra-industry trade in the same commodity and the tariff policy may derive a potential domestic entrant to produce both for the home and the foreign markets.

#### 1.2.4 Time Consistency

Kydland and Prescott (1977) define the notion of “time consistency.” An agreed-upon social objective function at time  $t$  is represented by a function of a sequence of policies and the corresponding sequence for economic agent’s decisions for periods 1 to  $T$ . Agent’s decisions in period  $t$  depend on all policy decisions and their past decisions. A policy is consistent, if for each time period  $t$ , a policy at time  $t$  maximizes its social objective function, taking as given previous agents’ decisions and that future policy decisions are similarly selected.

Staiger and Tabellini (1987) consider the case of a country where free trade is the Pareto optimal policy. The government pursues free trade to achieve the optimal allocation of labor, but then to surprise the workers with a tariff that improves the distribution of income by raising wages in the import-injured industry. Since, then, the marginal benefit of an unexpectedly higher tariff exceeds its marginal cost, the original announcement of a free trade policy is not time-consistent. Workers will then recognize that the policy commitment to free trade is not credible. Workers will come to expect the tariff and fewer will leave the import-threatened industry after a price decline. Yet, the very fact that they do not leave will make the tariff even more necessary on income-distribution grounds, to prevent wages in the industry from falling even below the level that could be earned as unskilled labor in other industries. The inability of a country to precommit to a free trade policy becomes the reason for protection. A commitment to a simple set of trading rules, which preclude the idea of the government attempting or the industry expecting a policy surprise, may often be superior to an activist but discretionary trade policy.

Lapan (1988) investigates the time inconsistency problem in production with

a time lag. When there is a time lag in production, the optimal tariff announced by the importing country before production of a foreign firm is not time consistent because the *ex post* elasticity of export supply is different from the *ex ante* elasticity. The time-consistent (*ex post*) tariff equilibrium is Pareto inferior to the precommitment equilibrium. He illustrates that if the importing (large) country taxes domestic production of importables, then all countries in trade can gain.

Maskin and Newbery (1990) extend time consistency into two period dynamics. (The previous two articles discuss, in fact, time consistency in a multi-stage, one period model.) They consider dynamic inconsistency of tariff equilibrium in a two-period model. In comparing the dynamically consistent equilibrium with the commitment equilibrium, the large importing country will set a lower tariff on the imported oil, and in the future will import less oil than if it were able to commit itself in advance to future tariff levels. If the country places relatively greater weight on future consumption, the results reverse, and the importer will be worse off than were it unable to impose tariffs at all. This dynamic time-inconsistency will be avoided as follows: If either oil is costly to extract or to store, then storage can be at best partially successful. If both costs are zero, storage restores the dynamic consistency of the precommitment equilibrium.

#### 1.2.5 Effects of a ratio VER

Mai and Hwang (1989, MH, henceforth) show that the (non) equivalence of tariffs and quotas depends on the particular value of the conjectural variations parameter, and the target ratio of imports to domestic production, and that the price under a ratio quota is always higher than that under a volume quota.

Harris (1985) demonstrates that, under a Bertrand–Nash game, VER imposed on the foreign firm is ‘voluntary’ because the imposition of VER will raise profits to both the foreign and the domestic firm producing the substitute goods. Assuming the demand for the foreign good does not exceed the VER, the paper asserts that the domestic firm takes the position of Stackelberg leadership, the foreign firm’s being a price follower. The game structure is altered due to the imposition of the VER by the domestic government because “the presence of the VER means that any increase in its own price must be met by an appropriate price increase by the foreign firm so that the VER is met” (p. 804). The major conclusion is that 1) VER facilitates price leadership by the protected domestic firms and lead to an increase in profits and prices for both the domestic and the foreign firm, 2) VER induces contraction of the domestic output, 3) VER is welfare reducing, and henceforth 4) VER may be a highly undesirable form of protection.<sup>3</sup>

Itoh and Ono (1984)<sup>4</sup> employ a Bertrand duopoly model with heterogeneous goods to examine the nonequivalence of tariffs and quotas. Their conclusions are as follows: 1) in the presence of VER, both producers earn greater profits by setting higher prices under home leadership than under foreign leadership, and 2) the domestic prices are higher under a quota than under the equivalent tariff whichever producer becomes a price leader under the tariff.

Assuming that two firms play simultaneously before and after the imposition of a quota on the foreign firm, Krishna (1985, 1989) analyzes the effects of a volume

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<sup>3</sup>Yano (1989) employs an intertemporal model to show that VER may not be a highly undesirable form of protection.

<sup>4</sup>Itoh and Ono (1982) discuss the same topic, but in a different model using a homogeneous good and Stackelberg leadership under Bertrand competition to yield the nonequivalence between tariffs and quotas.

quota under Bertrand–Nash equilibrium. The effects of the restriction depend on whether imports are substitutes or complements.<sup>5</sup> In the former case, no equilibrium exists in pure strategies, but there exists a unique mixed strategy equilibrium, where the domestic firm randomizes its production, while the foreign firm does not. As a result, VER facilitates collusion to raise prices and profits of both firms. The paper also shows that 1) tariffs and quotas are nonequivalent if goods are substitutes, 2) the profits of the domestic firm are higher under the VER than under the equivalent tariff, 3) the VER is preferred by the foreign firm to no restriction, and 4) the foreign firm would prefer the VER to the equivalent tariff, even if the tariff revenues were returned to it as a lump sum.

### 1.3 Objectives

This dissertation discusses three issues on the strategic trade policy: Precommitment, time consistency, and the effects of a ratio quota under Cournot–Nash equilibrium.

Two precommitment models are studied in Chapter 2: A Spencer–Brander model and a true Stackelberg model. A government is assumed to act first against firms, which behave as Cournot competitors against each other. The three-stage game is characterized as follows: i) a government chooses optimal investment and export subsidies in the first stage of a game, ii) in the second stage, firms make decisions on the

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<sup>5</sup>Krishna (1985, 1989) is different from Harris (1985), and Itoh and Ono (1984) in that the latter two papers consider only substitute goods and Stackelberg leadership. Harris is different from Itoh and Ono in that they consider Stackelberg leadership of the domestic and the foreign firm, while Harris considers Stackelberg leadership of only the domestic firm.



level of investment based on the announcement, and iii) in the last stage, firms choose the output levels. We show that firms choose “overinvestment,” “underinvestment,” or a level of investment that minimize costs, all depending on the responsiveness of the slope of the marginal cost due to a change in the level of the investment. We also show that precommitting both investment and output levels (i.e., true Stackelberg precommitment) yields a solution superior to the outcome described above.

Both precommitment models (Spencer–Brander and Stackelberg precommitment models) are not time consistent. We show how to derive a time consistent solution employing a four-stage game. We show, heuristically and via simulation, that the time consistent solution may be superior to the precommitment solutions, a result that is at odds with the conventional wisdom concerning the inferiority of the time consistent solution.

Few papers have analyzed the effects of a **proportional** VER. We investigate a case that the imposition of a ratio quota by the domestic government confers Stackelberg leadership on the home firm, the foreign firm’s being a Stackelberg follower. If this is the case, then the quota may or may not bind: It does not always bind unlike Mai and Hwang (1989). However, the assumption that the presence of the quota changes the game structure is in general unwarranted; the implementation of a ratio quota does not convey a first play advantage to the home firm. For example, under the steel agreements, market demand is forecast for the coming period, and then foreign firm import levels are assigned (as percents of this forecast). Given this forecast, domestic and foreign firms act simultaneously. Furthermore, if actual imports exceed the target share, a “penalty” is imposed by adjusting permissible imports for the subsequent period. Under this implementation, which is customary,

the strategic behavior of firms is most appropriately modeled as simultaneous play, not as Stackelberg leadership for the home firm. Simultaneous play entails that 1) there is no equilibrium in pure strategies, and 2) there exists a unique mixed strategy of the domestic firm's randomization of the production. When a tariff and a ratio quota yield the same profits to the home firm, the foreign firm prefers a quota to a tariff.

## 2. INVESTMENT AND EXPORT SUBSIDIES UNDER THE PRECOMMITMENT MODEL

### 2.1 Introduction

A significant proportion of international trade takes place in imperfectly competitive markets. The determinants and patterns of trade in these markets are evidently different from those in the perfect competition, upon which orthodox trade theory is based.

It is well known that firms under oligopolistic competition in general earn positive pure profits (sometimes called "super-normal" profits). Governments in many countries pursue various policy instruments such as export (production, or consumption) subsidies, taxes, or import tariffs, and one purpose of these policies may be to shift some rents toward its own country at the expense of a competing country. Many authors have recently developed models of oligopolistic trade theory, which provides the basis for a strategic trade policy.<sup>1</sup>

Spencer and Brander (S-B, hereafter) (1983), among others,<sup>2</sup> have pioneered this area to show, using two- or three-stage game-theoretic models, if the government

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<sup>1</sup>See *Strategic Trade Policy and the New International Economics* (1986) edited by Krugman.

<sup>2</sup>See Dixit (1984), Krugman (1984), and Brander and Spencer (1984 and 1985), and Eaton and Grossman (1986), Helpman and Krugman (1989), etc.

can act first, then its action will change final outcomes from those played by firms. Assuming a structure in which research and development (R & D) decisions are made before production decisions, and in which two firms, located in different countries, act as Cournot competitors at each stage of a game, S-B analyze how a country can use R & D subsidies and/or export subsidies (when feasible; they are prohibited by GATT, though) to improve domestic welfare. They show that if only R & D subsidies or only export subsidies are available, it is optimal to choose a positive level for that policy. However, they also show that when both policy instruments are available, both governments will tax R & D, and exports will be subsidized.

In this chapter, we first show that their optimum subsidies/taxes on export and R & D, which are simultaneously determined at the first stage of a game, are miscalculated, and that in general it will not be optimal to choose an R & D tax that induces the domestic firm to choose that investment or R & D level that minimizes costs. Secondly, we expand the S-B analysis by considering a model in which the active government is a full Stackelberg leader with respect to both R & D and output, and we compare this equilibrium to that of S-B: When a country takes a full Stackelberg leader position, the welfare of that country will be increased.

Furthermore, no papers concerning the oligopolistic trade consider time consistency of optimal policies.<sup>3</sup> For example, if a government announces an optimal export subsidy before a firm builds a plant (or invests capital), the export subsidy that appears optimal once the plant has been constructed differs from that which

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<sup>3</sup>The notion of "time consistency" was introduced by Kydland and Prescott (1976). Under perfect competition, time consistency is explained to be one cause of trade by Staiger and Tabellini (1987), and Lapan (1988) shows that time consistency solution is Pareto inferior to precommitment, and that *ex ante* production tax of an importing country will boost welfare of both countries.

is optimal from an *ex ante* perspective, and hence it is unlikely that a government will keep the export subsidy at its pre-announced level. Thus, a problem of time inconsistency arises when a government attempts to predetermine its export subsidy in this multi-stage game. We compare the responsiveness of the output to a change in capital, the level of the capital investment, and welfare between the Spencer and Brander and the Time Consistency regimes in Chapter 3.

In this chapter, we employ essentially the same model as in the S-B analysis. We assume that two rival firms, located in two different countries, compete only in a third country to focus on the rent-seeking behavior of each firm. The importing country does not impose a tariff, and only one exporting country (call it the domestic country) is assumed to provide capital and export subsidies. The two firms act as Cournot competitors, but may (in the time consistent solution) recognize and capitalize on the endogeneity of government policy.

We briefly describe, in the next section, the nature of the S-B paper, especially, on the game stage by stage, and on the simultaneous determination of R and D, and export subsidies. It will be shown that the S-B computation of the subsidies is wrong, and the correct calculation will be derived. Even these optimum subsidies, however, will not get the full Stackelberg solution. In the third section, we will explain the nature of the Stackelberg game in this regime, show how it can be supported, and compare welfare of the S-B with that of the Stackelberg solution. Time consistency will be presented in Chapter 3.

## 2.2 The Spencer–Brander Model

The pre-commitment model (S–B) is characterized by the three stage game: i) in the first stage, a domestic government announces export and capital subsidies at the same time, ii) in the second stage, firms choose levels of capital investment, and iii) in the third stage, firms choose output levels. We solve this time sequence with backward induction as the S–B paper does. As such, we begin our analysis from the last stage of choosing the outputs to maximize the profits of each firm.

### 2.2.1 The basic model

Let  $B$  and  $\bar{B}$  denote profits (net of any subsidies) of a domestic and foreign firm, respectively, such that

$$B = qp - c(q, k), \text{ and } \bar{B} = \bar{q}p - \bar{c}(\bar{q}, \bar{k}), \quad (2.1)$$

where  $q$  and  $\bar{q}$  are outputs (sales) of the home and foreign firms,<sup>4</sup>  $p$  is a price both firms receive from a unified market (a third country), which is a function of  $q + \bar{q}$ ,  $c$  ( $\bar{c}$ ) is a reduced cost function of each firm, and  $k$  ( $\bar{k}$ ) is a level of capital investment in both countries.<sup>5</sup>

When a home government supports her domestic firm by subsidizing capital investment and export at rates of  $s$  and  $z$  per unit, respectively, profit of the home firm becomes  $B + sk + zq$ .

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<sup>4</sup>The barred variables denote foreign firm's.

<sup>5</sup>Our capital investment ( $k$ ) corresponds to R & D ( $x$ ) in S–B. The function,  $c(q, k)$ , in this chapter embodies the fixed per unit cost ( $v$ ) of R & D assumed in the S–B paper. The models are formally identical.

In the last stage of the game, each firm chooses its own output,  $q$  ( $\bar{q}$ ), to maximize profits. The first order conditions (FOCs) are:

$$B_q + z = 0, \text{ and} \quad (2.2)$$

$$\bar{B}_{\bar{q}} = 0, \quad (2.3)$$

assuming second order conditions ( $B_{qq} < 0$ , and  $\bar{B}_{\bar{q}\bar{q}} < 0$ ) are satisfied. The first equation yields the reaction function of the domestic firm ( $q = \phi(\bar{q}, k, z)$ ), the second one the reaction function of the foreign firm ( $\bar{q} = \bar{\phi}(q, \bar{k})$ ) in output space. Solving simultaneously, we obtain the optimal outputs of both firms in terms of capital investment and export subsidy such that:

$$q = h(k, \bar{k}, z), \text{ and} \quad \bar{q} = \bar{h}(k, \bar{k}, z). \quad (2.4)$$

Totally differentiating the FOCs with respect to  $q$ ,  $\bar{q}$ ,  $k$ ,  $\bar{k}$ , and  $z$  yields the responsiveness of outputs to investment and subsidy decisions:

$$\begin{pmatrix} B_{qq} & B_{q\bar{q}} \\ \bar{B}_{\bar{q}q} & \bar{B}_{\bar{q}\bar{q}} \end{pmatrix} \begin{pmatrix} dq \\ d\bar{q} \end{pmatrix} = \begin{pmatrix} -B_{qk}dk - dz \\ -\bar{B}_{\bar{q}\bar{k}}d\bar{k} \end{pmatrix}. \quad (2.5)$$

We assume the determinant ( $A$ ) of the coefficient matrix above is positive for the stability and uniqueness of the system:  $A = B_{qq}\bar{B}_{\bar{q}\bar{q}}(1 - xy) > 0$ , where  $x = \bar{B}_{\bar{q}q}/\bar{B}_{\bar{q}\bar{q}}$ , and  $y = B_{q\bar{q}}/B_{qq}$ . The term  $-x$  ( $-y$ ) is the slope of the reaction function of a domestic (foreign) firm in output space, and is assumed in general to be negative. (i.e., cross-partials of each firm's profit function are negative as well.) For capital, "normality" is assumed; that is,  $B_{qk} = -c_{qk}$  is positive. The normality assumption

is interpreted as follows: As the level of capital investment rises, marginal cost falls. (This is true for both firms.)

Inverting the matrix equation (1.5) yields:

$$\begin{pmatrix} dq \\ d\bar{q} \end{pmatrix} = \frac{1}{A} \begin{pmatrix} +\bar{B}_{\bar{q}\bar{q}} & -B_{q\bar{q}} \\ -\bar{B}_{\bar{q}q} & +B_{qq} \end{pmatrix} \begin{pmatrix} -B_{qk}dk - dz \\ -\bar{B}_{\bar{q}\bar{k}}d\bar{k} \end{pmatrix}. \quad (2.6)$$

Comparative static results are interpreted as follows: With a given level of export subsidy, an increase in  $k$ , which does not affect the reaction function of the foreign firm, will shift outward the reaction function of the domestic firm, causing domestic output ( $q$ ) to rise, and foreign output ( $\bar{q}$ ) to fall. Similar results hold for  $\bar{k}$ . Mathematically:

$$\begin{aligned} \partial q / \partial k \equiv h_k &= -B_{qk} \bar{B}_{\bar{q}\bar{q}} / A > 0, \\ \partial q / \partial \bar{k} \equiv h_{\bar{k}} &= -y \bar{h}_{\bar{k}} < 0, \end{aligned} \quad (2.7)$$

$$\begin{aligned} \partial \bar{q} / \partial k \equiv \bar{h}_k &= -x h_k < 0, \\ \partial \bar{q} / \partial \bar{k} \equiv \bar{h}_{\bar{k}} &= -B_{q\bar{q}} \bar{B}_{\bar{q}\bar{k}} / A > 0. \end{aligned} \quad (2.8)$$

At a given levels of  $k$  and  $\bar{k}$ , the export subsidy in one country leads to an increase in its own output and a decrease in the output of its rival:

$$\partial q / \partial z = h_z = -\bar{B}_{\bar{q}\bar{q}} / A > 0, \text{ and } \partial \bar{q} / \partial z = \bar{h}_z = -x h_z < 0. \quad (2.9)$$

Knowing the rules in output space (especially the FOCs above), firms play a game at the preceding stage by choosing  $k$  and  $\bar{k}$  to maximize profit, where the profit functions of each firm will be defined as; for the home firm,

$$g(k, \bar{k}, z; s) = B(h, \bar{h}, k) + zh + sk, \quad (2.10)$$



and, for the foreign firm,

$$\bar{g}(k, \bar{k}, z) = \bar{B}(h, \bar{h}, \bar{k}). \quad (2.11)$$

Choice of  $k$  by the home firm entails:

$$g_k = (B_q + z)h_k + s + B_k + B_{\bar{q}}\bar{h}_k = s + B_k + B_{\bar{q}}\bar{h}_k = 0, \quad (2.12)$$

where the first term is zero by the envelope theorem ( $B_q + z = 0$ ).

Choice of  $\bar{k}$  by a foreign firm gives us:

$$\bar{g}_{\bar{k}} = \bar{B}_{\bar{q}}\bar{h}_{\bar{k}} + \bar{B}_q h_{\bar{k}} + \bar{B}_{\bar{k}} = \bar{B}_q h_{\bar{k}} + \bar{B}_{\bar{k}} = 0, \quad (2.13)$$

where the first term is zero by the envelope theorem ( $\bar{B}_{\bar{q}} = 0$ ).

Both equations represent reaction functions of each firm in input space. A capital subsidy,  $s$ , by a domestic country shifts outward the reaction function of its firm, while that of the foreign firm stays the same, to cause  $k$  to increase and  $\bar{k}$  to decrease. An export subsidy,  $z$ , affects  $k$  and  $\bar{k}$  indirectly through the impact on outputs and the responsiveness of output to investment.

For future reference, note that  $\bar{g}_{\bar{k}} = 0$  defines an implicit function in  $(k, \bar{k}, z)$ :

$$M(k, \bar{k}, z) \equiv \bar{g}_{\bar{k}} = \bar{B}_q h_{\bar{k}} + \bar{B}_{\bar{k}} = 0. \quad (2.14)$$

Notice that the equation (2.14) is same as the one (2.13). By the SOC,  $M_{\bar{k}} < 0$ .

Following S-B, assume:

$$M_k = \{[\bar{B}_{qq} + \bar{B}_{q\bar{q}}\bar{\phi}_q]h_{\bar{k}} + \bar{B}_{\bar{k}\bar{q}}\bar{\phi}_q\}h_k + \bar{B}_q h_{\bar{k}k} < 0, \quad (2.15)$$

which implies the reaction functions are negatively sloped.<sup>6</sup> Similarly, assume:

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<sup>6</sup>For stability and uniqueness, we assume that  $g_{kk}\bar{g}_{\bar{k}\bar{k}} - g_{k\bar{k}}\bar{g}_{k\bar{k}} > 0$ . We also assume that the slope of each reaction function is negative and greater than  $(-1)$ ; e.g.,  $0 > \partial\bar{k}/\partial k = -M_k/M_{\bar{k}} > -1$ .

$$M_z = \{[\bar{B}_{qq} + \bar{B}_{q\bar{q}}\bar{\phi}_q]h_{\bar{k}} + \bar{B}_{\bar{k}\bar{q}}\bar{\phi}_{\bar{q}}\}hz + \bar{B}_qh_{\bar{k}z} < 0, \quad (2.16)$$

which implies that an increase in  $z$  shifts the foreign firm's reaction function inward (i.e., an increase in  $z$ , given  $k$ , decreases  $\bar{k}$ ).

A domestic government, as a first mover in this game, chooses the optimal subsidies on capital investment and export before investment occurs, given the rules of equations (2.2), (2.3), (2.12), and (2.14).<sup>7</sup> The export subsidy affects not only the choice of output given R & D level, but R & D directly. In their paper, S-B show that the optimal export subsidy is positive, but that R & D should be taxed. They also claim to show that the optimal R & D tax achieves cost minimization (i.e.,  $B_k = 0$ ).

We show in what follows that, in general,  $B_k \neq 0$ , and hence the optimum R & D subsidy may be difficult to sign.

### 2.2.2 Optimal government instruments

The optimization of welfare function ( $B$ ) over  $s$  and  $z$ , given a rule of equations (2.2), (2.3), (2.12), and (2.14), by a home government at the first stage is equivalent to the choice of  $q$  and  $k$  to maximize  $B$ , given equations (2.3), and (2.14).<sup>8</sup>

Specifically, since  $q = h(k, \bar{k}, z)$  is monotone in  $z$ , it can be inverted to yield  $z(k, \bar{k}, q)$ . Substituting this into  $M$  yields  $\bar{k}$  in terms of  $k$  and  $q$ . Define:

$$L(k, \bar{k}, q) \equiv M(k, \bar{k}, z(k, \bar{k}, q)) = 0. \quad (2.17)$$

---

<sup>7</sup>This is what occurs at the first stage of the game. We solve it at this late stage by a nature of "backward induction."

<sup>8</sup>From (2.2),  $z$  can be determined as a function of  $q$ ,  $\bar{q}$ , and  $k$ . Similarly, from (2.12),  $s$  can be determined as a function of  $q$ ,  $\bar{q}$ ,  $k$ , and  $\bar{k}$ . Hence, the choice of  $s$  and  $z$ , given (2.2), (2.3), (2.12), and (2.14), is equivalent to choosing  $q$  and  $k$ , given (2.3) and (2.14).

Since  $M_z < 0$ , and from (2.6),  $z_{\bar{k}} = -h_{\bar{k}}/h_z > 0$ ,  $L$  is a decreasing function of  $\bar{k}$ . Inverting yields  $\bar{k} = \Omega(k, q)$ , where from (2.6), (2.16), and (2.17):

$$\frac{\partial \bar{k}}{\partial q} \equiv \Omega_q = -M_z z_q / L_{\bar{k}} < 0, \quad (2.18)$$

where  $L_{\bar{k}} = M_{\bar{k}} + M_z z_{\bar{k}} < 0$ .

$$\frac{\partial \bar{k}}{\partial k} \equiv \Omega_k = -(M_k + M_z z_k) / L_{\bar{k}} \quad (2.19)$$

$$= -\bar{B}_q \bar{\phi}_{\bar{k}} \phi_{\bar{q}k} / A^2 L_{\bar{k}}, \quad (2.20)$$

where  $\phi_{\bar{q}k} = \bar{B}_{q\bar{q}} B_{qqk} / B_{q\bar{q}}^2$ , and  $B_{qqk} = -c_{qqk}$ .

Intuitively, given  $q$ , changes in  $k$  (accompanied by offsetting changes in  $z$  which hold  $q$  constant) affect the foreign choice of  $\bar{k}$  only through their impact on  $h_{\bar{k}}$ , and hence through their impact on the slope of the domestic reaction function ( $\phi_{\bar{q}}$ ).

Thus, maximizing  $B$  over  $(q, k)$ , given (2.3) and (2.17) yields:

$$[B_q + B_{\bar{q}} \bar{\phi}_q] + B_{\bar{q}} \bar{\phi}_{\bar{k}} \Omega_q = 0, \quad (2.21)$$

and

$$B_k + B_{\bar{q}} \bar{\phi}_{\bar{k}} \Omega_k = 0. \quad (2.22)$$

Solving (2.3), (2.17), (2.21), and (2.22) yields the optimal solution  $(q^*, \bar{q}^*, k^*, \bar{k}^*)$ .

The optimal subsidies are found from (2.2) and (2.12):

$$z^* = B_{\bar{q}} [\bar{\phi}_q + \bar{\phi}_{\bar{k}} \Omega_q] > B_{\bar{q}} \bar{\phi}_q > 0, \quad (2.23)$$

because  $\bar{\phi}_{\bar{k}} \Omega_q < 0$ , and

$$s^* = B_{\bar{q}} [\bar{\phi}_{\bar{k}} \Omega_k - \bar{h}_k]. \quad (2.24)$$

From (2.22), it can be seen that the S-B claim that, at the optimal solution, domestic firms will choose the investment level that minimizes costs is, in general, incorrect.

*Proposition 1:*

At the optimal solution,  $B_k$  is greater than, equal to, or less than zero as  $c_{qqk}$  is less than, equal to, and greater than zero.

*Proof:*

From (2.20), and (2.22)

$$B_k = -B_{\bar{q}}\bar{\phi}_{\bar{k}}\Omega_k \quad (2.25)$$

$$= -(\bar{B}_q B_{\bar{q}} \bar{\phi}_{\bar{k}}^2 B_{q\bar{q}} / A^2 L_{\bar{k}} B_{qq}^2) c_{qqk}. \quad (2.26)$$

Since  $(\bar{B}_q, B_{\bar{q}}, B_{q\bar{q}}$  and  $L_{\bar{k}})$  are all negative, the proposition follows immediately.

This result implies that capital will be overinvested ( $B_k < 0$ ) if  $c_{qqk} > 0$ , underinvested if  $c_{qqk} < 0$ , and cost minimization will be achieved only when  $c_{qqk} = 0$ .

Intuitively, the reason that it is not, in general, optimal to minimize costs ( $c_k \neq 0$ ) is that domestic investment affects the foreign investment decision. As noted earlier, for given  $q$ , domestic investment affects foreign investment, and hence foreign output, only insofar as it affects the slope of the domestic reaction function in output space.

If  $c_{qqk} > 0$ , then increased domestic investment reduces the responsiveness of domestic output to foreign output (i.e., makes the domestic reaction function less steep,  $\phi_{\bar{q}k} > 0$ ) and hence reduces the foreign firm's incentive to overinvest. Thus,

for  $c_{qqk} > 0$ , it will be optimal for the domestic firm to overinvest (i.e.,  $B_k < 0$ ); converse results hold for  $c_{qqk} < 0$ . Only if  $c_{qqk} = 0$  (i.e., only if the slope of the domestic reaction function is independent of  $k$ ) is it optimal to minimize costs.<sup>9</sup>

From (2.24), the sign of the optimal capital subsidy/tax may depend on  $c_{qqk}$ . If  $c_{qqk} \leq 0$  ( $\phi_{\bar{q}k} \leq 0$ ), then it is optimal to tax capital, as in S-B. However, for  $c_{qqk} > 0$ , the sign of the optimal capital subsidy is indeterminate.

The solutions in this section (i.e., in the S-B model) are not fully optimal in that the model (and the instruments used) do not allow the government to fully exploit its first-play advantage. Once the government predetermines its investment and export subsidies, the domestic and foreign firms make investment decisions based upon these subsidies and upon the responsiveness of the other firm's output level to their investment decisions. As seen in equation (2.13) ( $\bar{B}_{qh\bar{k}} + \bar{B}_{\bar{k}} = 0$ ), this leads the foreign firm to overinvest in capital ( $\bar{B}_{\bar{k}} < 0$ ), thereby reducing domestic welfare. As we show in the next section, domestic welfare could be increased if the domestic government could devise a policy that allowed it (or its domestic firm) to precommit output and investment.

### 2.3 Full Precommitment

We show in this section that if the home country can precommit both its investment decision (as in S-B) and its output decision, then the resulting solution yields higher domestic welfare.

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<sup>9</sup>The error in the S-B paper occurs on page 720, line 1, where they assume  $q_{z2}^1 = -q_{12}^1/c_x^1$ . This result hold only if the slope of the domestic reaction function is independent of  $k$ : i.e., only if  $c_{yx}^1$  (using their notation) is zero. The proof is in Appendix A.

Thus, suppose, through some set of instruments, the home government chooses  $q$  and  $k$  before the foreign firm acts, but knowing the foreign firm's decision rules. Given  $q$  and  $k$ , the foreign firm chooses first  $\bar{k}$ , and subsequently  $\bar{q}$ , to maximize profits. Since  $q$  and  $k$  are predetermined, the sequential nature of the foreign firm's decisions is immaterial and the resulting first order conditions are:

$$\bar{B}_{\bar{q}} = 0, \text{ and} \quad (2.27)$$

$$\bar{B}_{\bar{k}} = 0. \quad (2.28)$$

Solving the equation (2.27) ( $\bar{B}_{\bar{q}} = 0$ ) for  $\bar{q}$  in terms of  $q$  and  $\bar{k}$  yields the foreign firm's short-run reaction function,  $\bar{q} = \bar{\phi}(q, \bar{k})$ , as earlier.<sup>10</sup> Solving the equation (2.27) and the equation (2.28) simultaneously yields the foreign firm's long-run reaction function,  $\bar{q} = \theta(q)$ . Its slope is given by:

$$d\bar{q}/dq \equiv \theta'(q) = \frac{-\bar{B}_{\bar{q}q}}{\bar{B}_{\bar{q}\bar{q}} - \bar{B}_{\bar{k}\bar{q}}^2/\bar{B}_{\bar{k}\bar{k}}}. \quad (2.29)$$

Comparing the above equation (2.29), the slope of the foreign firm's reaction function, to  $\bar{\phi}_q = \bar{B}_{\bar{q}q}/\bar{B}_{\bar{q}\bar{q}}$ , which is the slope of the foreign firm's short-run reaction function, it is apparent that the long-run reaction function is steeper: That is,  $|\theta'(q)| > |\bar{\phi}_q|$ . This relationship is shown in Figure 1, where SR (LR) denotes the short-run (long-run) reaction function of the foreign firm. Given this long-run reaction function, the domestic government chooses  $q$  and  $k$  to maximize domestic welfare.

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<sup>10</sup>It is a short-run reaction function in a sense that  $\bar{k}$  is treated as fixed, whereas in the long-run reaction function  $\bar{k}$  is endogenous.

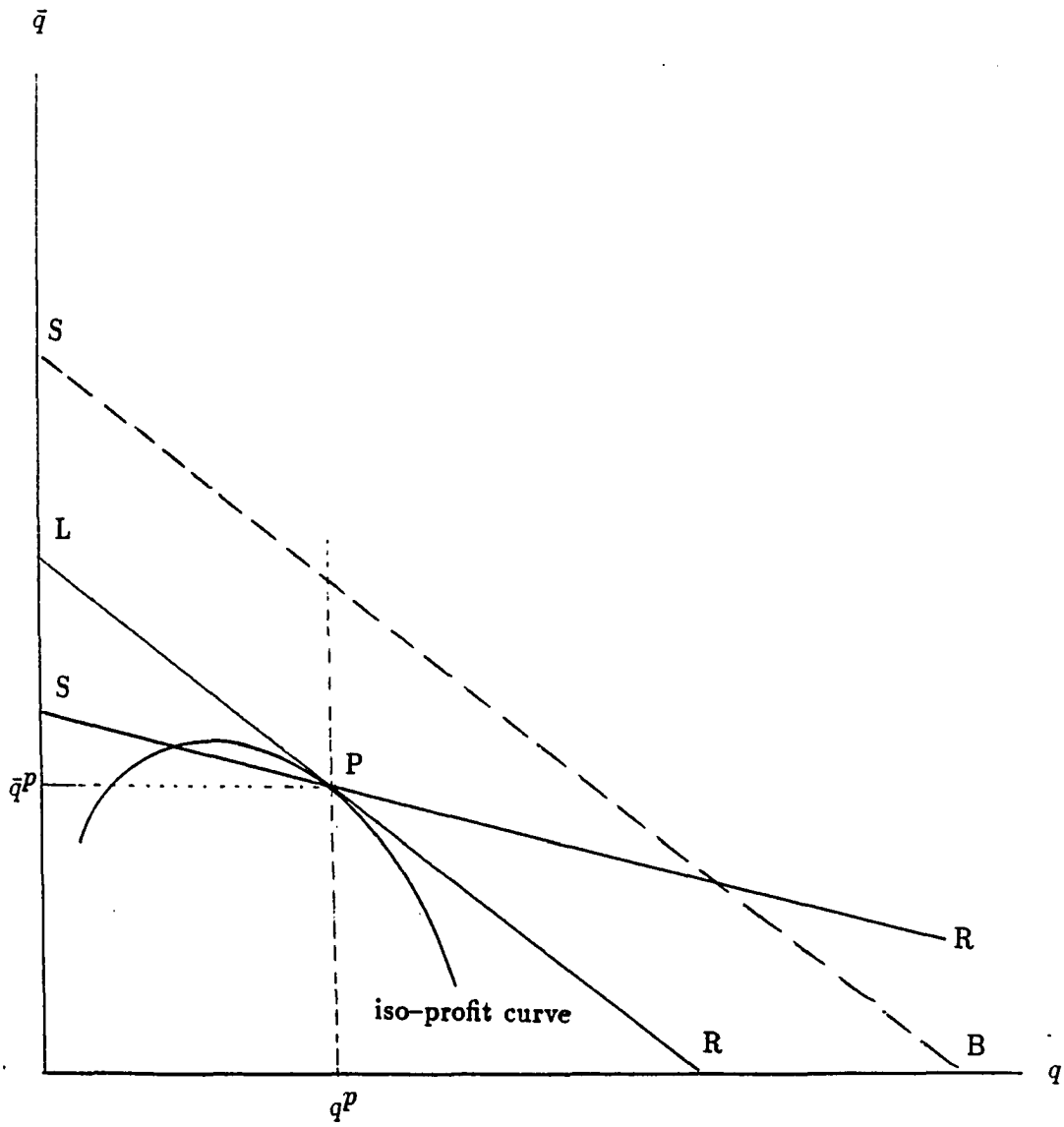


Figure 2.1: Short-Run and Long-Run Reaction Functions

Since  $\bar{q}$  is independent of  $k$ , the optimal solution entails cost minimization. Thus:

$$B_k = 0, \text{ and} \quad (2.30)$$

$$B_q + B_{\bar{q}}(d\bar{q}/dq) = 0. \quad (2.31)$$

Equations (2.27), (2.28), (2.30), and (2.31) determine the full-precommitment solution (which we label as  $q^P$ ,  $\bar{q}^P$ ,  $k^P$ , and  $\bar{k}^P$ ). This solution is depicted in Figure 1 by a tangency of the domestic firm's iso-profit curve with the long-run reaction function (at point P). Domestic investment ( $k^P$ ) is not shown explicitly, but is consistent with the level required to minimize production costs. Foreign investment ( $\bar{k}^P$ ) is chosen to minimize foreign costs, given output; the curve labeled SR passing through the point P shows the short-run foreign reaction function consistent with this level of investment.

### 2.3.1 Supporting the optimal solution

This solution cannot be supported by having the domestic government precommit to investment and export subsidy rates, but a variety of other instruments could be used to support this solution. For example, the domestic government could predetermine an export subsidy rate ( $z^*$ ) and an export quota ( $Q$ ), the latter being set at the optimal export level ( $q^P$ ); no investment (or R & D) subsidy is required to support this solution.

Given these instruments, in the last stage of the game, domestic and foreign firms choose output levels, given prior investment decisions and these policy instruments. The foreign firm's optimal rule is given, as earlier, by (2.3). The domestic firm chooses  $q$ , given  $k$ ,  $\bar{q}$ , and  $z$ , and the export quota ( $Q \geq q$ ), to maximize profits.



Its constrained objective function, denoted as  $G$ , and first order conditions are as follows:

$$G = B(q, \bar{q}, k) + z^*q + \lambda(Q - q), \text{ and} \quad (2.32)$$

$$G_q = B_q + z^* - \lambda \quad (2.33)$$

$$= 0, \quad (2.34)$$

where  $\lambda$  denotes the Lagrangean multiplier corresponding to the export quota ( $Q$ ) constraint. Solving ( $\bar{B}_{\bar{q}} = 0$ ) and ( $G_q = 0$ ) yields the optimal solutions for  $q$  and  $\bar{q}$  as functions of prior investment, the export subsidy (if the constraint is not binding), and the export quota (if it is binding). Given any prior investment decisions, there exists a subsidy rate  $\hat{z}(k, \bar{k}, Q)$  such that the export quota will be binding if  $z^* > \hat{z}$ . If the export quota is binding the solution for  $q$  is independent of  $k$  and  $\bar{k}$ , whereas the solution for  $\bar{q}$  is independent of  $k$ .

At the preceding stage, firms choose investment decisions, given the output rules determined above. The FOC's for the foreign and home firm are given as follows:

$$\bar{B}_{\bar{k}} + \bar{B}_q(dq/d\bar{k}) = 0, \text{ and} \quad (2.35)$$

$$B_k + (B_q + z^*)(dq/dk) + B_{\bar{q}}(d\bar{q}/dk) = 0. \quad (2.36)$$

In the latter equation, the middle term is zero since either the export constraint is binding ( $dq/dk = 0$ ), or else  $B_q + z^* = 0$ . Furthermore, if  $z^*$  is "large enough" so that the export constraint is binding at the precommitment solution ( $k^P$  and  $\bar{k}^P$ ), then both  $dq/dk$  and  $d\bar{q}/dk$  are zero.<sup>11</sup> Hence:

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<sup>11</sup>The export subsidy must be set strictly larger than  $\hat{z}(k^P, \bar{k}^P, q^P) = B_q(d\bar{q}/dq)$  evaluated at the precommitment solution, since the right hand derivative of  $q$  and  $\bar{q}$

*Proposition 2:*

The output and investment precommitment solution can be supported with an export subsidy and an export quota; no investment subsidy/tax is required.

While it may seem paradoxical to use both an export subsidy and an export quota, the intuitive explanation is straightforward. The subsidy, as in S-B, is needed to transform the domestic firm into a Stackelberg leader. The export quota, on the other hand, is useful because of the two-stage nature of the game, inasmuch as it—in conjunction with the subsidy—makes the foreign firm perceive that domestic output is not responsive to the foreign firm's investment decision.

It is somewhat ironic to note that many recent trade agreements have tended to use export VERs as an attempt to compensate for (alleged) subsidies received by these firms. The analysis here indicates that the combination of export subsidies and quotas can strengthen, not weaken, the position of countries which use strategic trade policy, and hence may hurt the very countries that have caused the VERs to be initiated.

#### 2.4 Comparing the S-B and Full Precommitment Solutions

As we have seen, either solution can be viewed as obtained by maximizing domestic welfare, over  $q$  and  $k$ , subject to the two constraints imposed by the behavior of the foreign firm. The constraints for the two separate problems differ only in at this point will be nonzero. Given the export quota, the level at which  $z^*$  is set is immaterial, provided it is sufficiently large since it only involves a transfer payment, with no production effects. Also note that in theory a 100% *ad valorem* export tax, combined with a specific subsidy, could be used to support this solution.

the equation reflecting the foreign firm's (first-stage) optimization over investment. Thus, let:

$$J = \bar{B}_{\bar{k}} + \epsilon \bar{B}_q h_{\bar{k}} = 0, \quad (2.37)$$

where  $J$  is a function of  $q$ ,  $k$ ,  $\bar{q}$ ,  $\bar{k}$ , and  $\epsilon$  with  $J_\epsilon > 0$  and  $J_{\bar{k}} < 0$ . Note that for  $\epsilon = 0$   $J$  reflects the constraint corresponding to the full precommitment solution, whereas  $\epsilon = 1$  corresponds to the S-B solution.

The optimization problem above is equivalent to maximizing a Lagrangean function,  $F$ , over  $q$  and  $k$  such that

$$F = B + \delta_1(-\bar{B}_{\bar{q}}) + \delta_2(-J), \quad (2.38)$$

where  $\delta_1$  and  $\delta_2$  are Lagrangean multipliers. Optimizing  $F$  over  $q$ ,  $k$ ,  $\delta_1$ , and  $\delta_2$  yields the solution as a function of  $\epsilon$ . It is readily shown  $\delta_1$  and  $\delta_2$  are positive.<sup>12</sup>

Let's denote an optimized Lagrangean function as  $F^*$ .<sup>13</sup> Then we have:

$$\frac{dF^*}{d\epsilon} = -\delta_2 h_{\bar{k}} \bar{B}_q < 0. \quad (2.39)$$

*Proposition 3:*

Welfare with a true Stackelberg solution exceeds that with a S-B regime.

This result can be illustrated using Figure 1. Given  $q$  and the rule  $\bar{B}_{\bar{q}}(q, \bar{q}, \bar{k}) = 0$  the value of  $\bar{k}$  that solves  $J = 0$  is an increasing function of  $\epsilon$ . Thus, as  $\epsilon$  increases,  $\bar{k}(q)$  increases; this, in turn, implies that given  $q$ , the value of  $\bar{q}$  that solves  $\bar{B}_{\bar{q}} = 0$

<sup>12</sup>Relaxing either constraint (i.e., permitting  $\bar{B}_{\bar{q}} > 0$  or  $\bar{B}_{\bar{k}} + \epsilon \bar{B}_q h_{\bar{k}} > 0$ ) will raise domestic welfare as it shifts the foreign reaction function toward the origin.

<sup>13</sup> $F^* = B^*$ .

increases; i.e., the foreign (long-run) reaction function shifts outward (to SB in Figure 1). Hence the original optimal point, P, is no longer feasible, and domestic welfare must fall.

The impact of increases in  $\epsilon$  on optimal (domestic and foreign) output and investment levels could be obtained by totally differentiating the first order conditions obtained from MAX F (which are not shown). Intuitively, we might expect that domestic output, and hence, investment, would be lower in the S-B case than in the full precommitment case because the S-B case encourages overinvestment by the foreign firm. Somewhat surprisingly, this need not be true. It can be shown that even in the simplest cases, the S-B case could lead to higher or lower domestic output and investment levels than the full precommitment case; similarly, it is possible –though less likely– that foreign output is lower in the S-B case than in the full precommitment case.<sup>14</sup>

How an increase in  $\epsilon$  affects domestic output depends upon how the slope of the foreign reaction function changes and how the slope of the domestic iso-profit (or iso-welfare) function changes as one moves outward (to higher  $\bar{q}$  at a given  $q$ ). However, there is simply no specific condition that allows one to sign these slope shifts, so no specific comparative static results are feasible.

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<sup>14</sup>For example, if demand is linear and the cost function is quadratic (so that the optimal solution entails cost-minimization in either case) then an analytic solution can be obtained. For this special case, we have found that, if foreign costs exceed domestic costs, then the S-B solution could result in higher domestic output than the full precommitment solution. Hence, it is futile to look for determinate comparative static results for general functional forms.

### 3. TIME CONSISTENCY

#### 3.1 Introduction

The S-B solution (or the full pre-commitment solution) presupposes that a government will not, or cannot, deviate from previously announced plans (export subsidies/quotas) once investment decisions are made. In reality, considerable time can elapse from the investment (or R & D) stage of the game to the time when output starts being exported. The supposition that the government will not change the promised level of export subsidies assumes that the government values its reputation. Yet, changes in governments or leaders are quite possible during this period. Even if the government continues to act in what it perceives to be the national interest, it is not necessarily committed to preserving the reputation of the preceding government. Hence, it may choose to re-evaluate the optimal export subsidy, in light of the previously determined levels of (domestic and foreign) investment.

Given investment, the optimal (*ex post*) export subsidy depends upon the slope of the short-run foreign reaction function, which is of necessity less than that of the long-run reaction function. As can be seen from (2.23), the *ex ante* subsidy is larger than that which appears optimal *ex post*. Hence, the S-B (or full precommitment) solution is **not** time consistent.

In the next section, we show how the optimal time consistent solution is deter-

mined. In Section 3, we compare welfare and output levels under the alternative solutions. Since no specific comparative static results are feasible, we employ simulation methods to show, for example, that a time consistent solution may dominate the alternatives in Section 4. Concluding remarks follow.

### 3.2 The Time Consistent Solution

The time consistent solution is characterized by the four-stage game: i) in the first stage, the domestic government announces the investment subsidy ( $s$ ), ii) in the second stage, firms choose levels of investment, given  $s$  and knowledge of how the time consistent export subsidy will be determined, iii) in the third stage, the domestic government sets the export subsidy given predetermined investment decisions of both firms, and finally, iv) in the fourth stage, firms choose output levels. Employing the backward induction method as in Chapter 2, we begin our analysis from the last stage of choosing the outputs to maximize the profits of each firm. The game at the last stage is the same as that in the S-B model in that both firms choose outputs to maximize their own profits. In the stage iv), firms choose  $(q, \bar{q})$ , given predetermined investment and export subsidy levels. The FOCs and short-run reaction functions (given below) are identical to those in (2.2)–(2.3). The superscript “t” denotes the time consistent solution. Thus;

$$B_q^t + z^t = 0, \text{ and } \bar{B}_{\bar{q}}^t = 0, \quad (3.1)$$

where the first and second equations give the reaction function of the home and foreign firms, respectively, in output space. The reaction functions of the domestic

and the foreign firm, respectively, are defined as:

$$q^t = \phi(\bar{q}^t, k^t, z^t), \quad \bar{q}^t = \bar{\phi}(q^t, \bar{k}^t). \quad (3.2)$$

From the above functions, one can, as earlier, solve for  $q^t(k^t, \bar{k}^t, z^t)$  such that:

$$q^t = h(k^t, \bar{k}^t, z^t), \quad (3.3)$$

which is the same functional form as that given by (2.4). In what follows it will be convenient to continue to use the short-run reaction function ( $\bar{\phi}$ ) for  $\bar{q}$ .

Given these rules and given  $(k^t, \bar{k}^t)$ , in the preceding stage, the home government chooses the optimal export subsidy to maximize welfare, net of subsidy expenditure. Using (3.1), this solution entails:

$$(B_q^t + B_{\bar{q}}^t \bar{\phi}_q) h z = 0. \quad (3.4)$$

We assume the SOC is satisfied such that  $q z^2 [B_{qq}^t + 2B_{q\bar{q}}^t \bar{\phi}_q + B_{\bar{q}\bar{q}}^t \bar{\phi}_q^2 + B_{\bar{q}}^t (\bar{\phi}_{qq} + \bar{\phi}_{q\bar{q}} \bar{\phi}_q)] < 0$ .

Letting  $x^t = -\bar{\phi}_q (= \bar{B}_{qq}^t / \bar{B}_{q\bar{q}}^t)$ , we have  $B_q^t - B_{\bar{q}}^t x^t = 0$ . We define  $z^t$  as follows:

$$z^t = -B_{\bar{q}}^t x^t = z(q^t, \bar{q}^t, \bar{k}^t), \quad (3.5)$$

where  $z_q = -(B_{qq}^t x^t + B_{q\bar{q}}^t x_{\bar{q}}^t)$ ,  $z_{\bar{q}} = -(B_{\bar{q}\bar{q}}^t x^t + B_{\bar{q}}^t x_{\bar{k}}^t)$ ,  $z_k = 0$ , and  $z_{\bar{k}} = -B_{\bar{q}}^t x_{\bar{k}}^t$ . Plugging the functions of  $q^t = h$  and  $\bar{q}^t = \bar{\phi}(h, \bar{k}^t)$  into the function  $z$  and rearranging it yield the solution:

$$z^t = \sigma(k^t, \bar{k}^t), \quad (3.6)$$

which is the time consistent export subsidy (for given investment levels). As can be seen from comparing (3.5) to (2.23), at the S-B level of investment  $(k^*, \bar{k}^*)$ , the

time consistent export subsidy is less than the export subsidy needed to support the S-B solution. Substituting the equation (3.6) into the equations (3.2)–(3.3) yields the time consistent output levels as functions of investment levels alone such that:

$$q^t = h(k^t, \bar{k}^t, \sigma(k^t, \bar{k}^t)) \equiv f^t(k^t, \bar{k}^t), \quad (3.7)$$

$$\bar{q}^t = \bar{\phi}(q^t(k^t, \bar{k}^t), \bar{k}^t) \equiv \bar{f}^t(k^t, \bar{k}^t). \quad (3.8)$$

Given the above equations, firms in the second stage choose  $(k, \bar{k})$  to maximize profits, recognizing the endogeneity of the export subsidy. The home and the foreign profit functions are given by:

$$g^t(k^t, \bar{k}^t, s^t) = B^t(f^t, \bar{f}^t, k^t) + f^t \sigma + s^t k^t, \quad (3.9)$$

$$\bar{g}^t(k^t, \bar{k}^t) = \bar{B}^t(f^t, \bar{f}^t, \bar{k}^t). \quad (3.10)$$

The FOCs for the home and the foreign firms are given by:<sup>1</sup>

$$g_k^t = B_k^t + B_q^t f_k^t + s^t + q^t \sigma_k = 0, \quad (3.11)$$

and

$$\bar{g}_{\bar{k}}^t = \bar{B}_{\bar{k}}^t + \bar{B}_q^t (h_{\bar{k}} + h_z \sigma_{\bar{k}}) = 0, \quad (3.12)$$

where  $f_{\bar{k}}^t = h_{\bar{k}} + h_z \sigma_{\bar{k}}$ .

In (3.11),  $s^t$  is the predetermined investment subsidy, and the last term ( $q^t \sigma_k$ ) reflects “rent-seeking” behavior as the domestic firm realizes that changes in investment alter the export subsidy (and transfer) it will receive from the government. In

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<sup>1</sup>In simplifying the FOCs, we use the envelope theorem,  $B_q^t + z^t = 0$  and  $\bar{B}_q^t = 0$ . We assume  $g_{kk}^t \bar{g}_{\bar{k}\bar{k}}^t - g_{k\bar{k}}^t \bar{g}_{\bar{k}k}^t > 0$ , so that the SOC is satisfied, and that  $(g_{k\bar{k}}^t, \bar{g}_{\bar{k}k}^t)$  are both negative, so that reaction functions in input space are both negatively sloped.



(3.12), we have separated out the total impact of foreign investment on domestic output ( $f_{\bar{k}}^t$ ) into that due to the shift in the foreign reaction function, for given  $z^t$  (which is  $h_{\bar{k}}$ ), and that due to the change in domestic output due to the change in the time consistent subsidy ( $hz\sigma_{\bar{k}}$ ).

Equation (3.12) determines foreign investment in terms of domestic investment:  $\bar{k}^t = \mu(k^t)$ , whereas (3.11) yields domestic investment in terms of  $(\bar{k}^t, s^t)$ . Since (3.11) is monotonic in  $k^t$  and  $s^t$ , it can be solved for  $s^t$  in terms of  $(k^t, \bar{k}^t)$ ; substituting  $\bar{k}^t = \mu(k^t)$  into this solution yields the domestic investment subsidy required to support any level of domestic investment.

Thus, the domestic government can be thought of as choosing  $k^t$  to maximize domestic welfare, subject to (3.7), (3.8), and (3.12). The FOCs for the optimal domestic investment are:

$$B_k^t + [B_q^t + B_{\bar{q}}^t \bar{\phi}_q][f_k^t + f_{\bar{k}}^t \mu'] + B_{\bar{q}}^t \bar{\phi}_{\bar{k}} \mu' = 0. \quad (3.13)$$

From (3.4), the second term on the LHS is zero. Since the export subsidy is time consistent, the choice of domestic investment depends only upon cost considerations ( $B_k^t$ ) and upon the change in foreign output due to the change in foreign investment induced by the change in domestic investment (the term  $\bar{\phi}_{\bar{k}} \mu'$ ). In other words, since the *ex post* domestic iso-welfare curve is tangent to the foreign *ex post* reaction function, it is only the shift in that reaction function which matters. Rewriting (3.13), using the envelope theorem:

$$B_k^t + B_{\bar{q}}^t \bar{\phi}_{\bar{k}} \mu' = 0 \quad (3.14)$$

Assuming that the foreign input reaction function is negatively sloped ( $\mu' < 0$ ), we have:

*Proposition 1:*

In the optimal time consistent solution, capital will be overinvested: i.e.,  $B_k^t < 0$ .

The proof follows trivially, given  $\mu' < 0$ . This overinvestment is dual to the result that the time consistent export subsidy is lower than the S-B subsidy. Since the domestic government cannot credibly use the threat of a large export subsidy to deter foreign investment, it must do so through higher investment subsidies: i.e., through overinvestment.

Equation (3.14), in conjunction with the earlier equations, determines the optimal time consistent solution: denote it by  $(\hat{k}, \hat{k}, \hat{q}, \hat{q})$ . The optimal investment subsidy required to support this solution ( $\hat{s}$ ) is obtained from (3.11) and (3.14) at the first stage of the game:

$$\hat{s} = -\hat{q}\sigma_k + B_{\hat{q}}[\bar{\phi}_{\hat{k}}\mu'(\hat{k}) - \bar{f}_{\hat{k}}], \quad (3.15)$$

where the first and third terms are negative, whereas the middle term is positive, so that  $\hat{s}$  cannot be signed. Our simulation results show that, depending upon parametric values,  $\hat{s}$  can be either positive or negative.

### 3.3 Comparison of S-B and Time Consistent Solutions

As for our earlier results concerning the S-B and full precommitment solution, no analytical comparisons of the solutions for the S-B and time consistent problems are possible. Based upon simulations we have performed, output and investment levels (for either domestic or foreign firms) can be larger under either model, depending upon specification of functional forms and choice of parametric values. Similarly,

the optimal investment subsidy can be negative (a tax) or positive, again depending upon the specification.

Perhaps more surprisingly, the welfare ranking (from the perspective of the domestic economy) of the time consistent solution and the S-B (or the full precommitment) solution also depends upon the model specification. That is, as we show in the next section, **it is possible that the time consistent solution is superior**, from the perspective of the domestic country, to either precommitment model considered in Chapter 2.

Before presenting these simulation results, it is instructive to see why this ambiguity arises. Let  $(\acute{q}, \acute{k}, \acute{\bar{q}}, \acute{\bar{k}}, \acute{z}, \acute{s})$  denote the optimal time consistent solution, where  $\acute{z} = \sigma(\acute{k}, \acute{\bar{k}})$  is the time consistent export subsidy. Assume that the domestic government can precommit its export subsidy, and that it can support any level of domestic investment through an appropriate investment subsidy. Let  $(k^b, z^b)$  denote the (*not necessarily optimal*) investment level and export subsidy to which the domestic government precommits, and choose  $k^b = \acute{k}$ , and  $z^b = \acute{z}$ . Then, the resulting equilibrium for this precommitment rule will differ from that of the optimal time consistent solution only if foreign investment differs under the two regimes. Let  $\bar{k}^b = \tau(k^b, z^b)$  denote optimal foreign investment under this regime, as determined from (2.15); and let  $q^b = h(k^b, \bar{k}^b, z^b)$ , and  $\bar{q}^b = \bar{h}(k^b, \bar{k}^b, z^b)$  denote the resulting outputs. Assuming capital is normal: As  $\bar{k}^b$  is less than, equal to, or greater than  $\acute{\bar{k}}$ ,  $q^b$  ( $\bar{q}^b$ ) is greater than, equal to, or less than  $\acute{q}$  ( $\bar{q}^b$ ).

For the time consistent solution  $\acute{\bar{k}}$  is determined by (3.12), whereas the precommitment solution  $\bar{k}^b$  is determined by (2.13). Evaluating (2.13) at  $\bar{k} = \acute{\bar{k}}$  yields;  $\bar{g}_{\bar{k}}(\acute{k}, \acute{\bar{k}}, \acute{z}) = -\bar{B}_q h_z \sigma_{\bar{k}}$ , which is greater than, equal to, or less than zero as  $\sigma_{\bar{k}}$  is

greater than, equal to, and less than zero.

Thus, if the optimal time consistent subsidy ( $\sigma(k, \bar{k})$ ) is a decreasing function of foreign investment ( $\sigma_{\bar{k}} < 0$ ), then the optimal foreign investment under the precommitment rule ( $k^b = \hat{k}, z^b = \hat{z}$ ) will be lower than under the time consistent solution ( $\bar{k}^b < \hat{k}$ ). This immediately implies domestic welfare will be higher under this precommitment solution, and *a fortiori* higher under the optimal S-B solution.

*Proposition 2:*

If the optimal time consistent export subsidy is a decreasing function of foreign investment, the S-B solution dominates the time consistent solution.

Clearly, this also implies that the time consistent solution is inferior to the full precommitment solution. Note, however, that this still does not allow one to compare optimal output and investment levels under the two models.

From (3.5),  $\hat{z} = \sigma(\hat{k}, \hat{k}) = p'q\bar{\phi}_q$ , where  $p'$  is the slope of the inverse demand curve. As  $\bar{k}$  increases,  $q$  will decrease, implying *ceteris paribus* that the optimal subsidy will decrease. **However**, the slope of the reaction function (and of  $p'$ ) may also change with  $\bar{k}$ . Assuming linear demand, how  $\bar{\phi}_q$  changes with  $\bar{k}$  depends upon the third derivatives of the cost function ( $\bar{c}_{\bar{q}\bar{q}\bar{q}}$  and  $\bar{c}_{\bar{q}\bar{q}\bar{k}}$ ). Thus, if demand is linear and costs are quadratic,  $\bar{\phi}_q$  is a scalar, and the S-B solution must dominate. However, for general cost structures no such conclusions are possible.

Similarly, the full precommitment solution must dominate the time consistent solution if  $f_{\bar{k}}^t < 0$ ; i.e., if the time consistent domestic output is a decreasing function of foreign investment. The condition that  $f_{\bar{k}}^t < 0$  is weaker than  $\sigma_{\bar{k}} < 0$ , so it is possible for the full precommitment solution to dominate the time consistent solution,

which in turn may dominate the S-B solution.

Demonstrating sufficient conditions under which the time consistent solution dominates the S-B solution is more difficult. If domestic and foreign investment levels under some time consistent solution are the same as under the optimal S-B solution, then the time consistent solution will dominate since the export subsidy will be chosen optimally (conditioned on these investment levels). Thus, a sufficient condition for the time consistent solution to be superior to the S-B solution is that  $(\partial \bar{g}^t / \partial \bar{k})$ , as given by (3.12), be non-positive when evaluated at  $(k = k^*, \bar{k} = \bar{k}^*)$ . However, the condition,  $\partial z^t / \partial \bar{k} \geq 0$ , is not sufficient to guarantee this result since the resulting output levels will differ under the two solutions. In general, it does not appear possible to establish analytical conditions which ensure that the time consistent solution is superior, so we rely on simulation results.

The main point is that none of the precommitment solutions are fully optimal since the export subsidy is not contingent on foreign investment levels, whereas the time consistent export subsidy is implicitly contingent on foreign investment. If the model structure is such that this time consistent subsidy is an increasing function of foreign investment, then the time consistent solution may be superior.

### 3.4 Comparing Alternative Solutions

In this section, we provide some simulation results to illustrate the points made in this chapter and Chapter 2. Suppose (home and foreign) firms have the following cost structure:

$$TC = cq + \beta(q - mk)^2/2 + w[q^2/2k + m^2k/2 - mq], \quad (3.16)$$

where  $(c, \beta, m, w)$  are nonnegative parameters; bars over the parameters will be used to represent the foreign firm.<sup>2</sup> If  $k$  is chosen to minimize costs, then LR marginal costs are constant ( $= c$ ), and optimal investment is given by  $mk^* = q$ .

Demand is assumed linear with intercept  $E$  and slope  $-1$ , so the normalized profit function for each firm is given by:

$$\begin{aligned}\pi &= (\alpha - \bar{q} - q)q - \beta(q - mk)^2/2 - w[q^2/2k + m^2k/2 - mq] + zq + sk, \\ \bar{\pi} &= (\bar{\alpha} - \bar{q} - q)\bar{q} - \bar{\beta}(\bar{q} - \bar{m}\bar{k})^2/2 - \bar{w}[\bar{q}^2/2\bar{k} + \bar{m}^2\bar{k}/2 - \bar{m}\bar{q}],\end{aligned}\quad (3.17)$$

where  $\alpha = E - c$ ,  $\bar{\alpha} = E - \bar{c}$ , and  $(z, s)$  are the subsidies received by the home firm.

The short-run reaction functions are:

$$a_{11}q = b_0 + z - \bar{q}, \quad (3.18)$$

$$a_{22}\bar{q} = b_1 - q, \quad (3.19)$$

where  $a_{11} \equiv 2 + \beta + w/k$ ,  $a_{22} \equiv 2 + \bar{\beta} + \bar{w}/\bar{k}$ ,  $b_0 \equiv \alpha + mw + \beta mk$ , and  $b_1 \equiv \bar{\alpha} + \bar{m}\bar{w} + \bar{\beta}\bar{m}\bar{k}$ .

For  $w > 0$ , and  $\bar{w} > 0$ , the slopes of the short run reaction functions,  $1/a_{ii}$ , are increasing functions of  $k$  (for  $i = 1$ ), and of  $\bar{k}$  (for  $i = 2$ ). Solving these equations yields the functions  $h(k, \bar{k}, z)$ , and  $\bar{h}(k, \bar{k}, z)$  as follows as in the equation (2.4):

$$q = [a_{22}(b_0 + z) - b_1]/A, \quad (3.20)$$

$$\bar{q} = [a_{11}b_1 - b_0 - z]/A, \quad (3.21)$$

where  $A = a_{11}a_{22} - 1$ .

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<sup>2</sup>If  $\beta = 0$ , the cost function may be derived from a Cobb-Douglas type function with  $Q = K^{.5}L^{.5}$ . Thus,  $TC = WL + RK = wq^2/K + RK$ , so that the parameter  $m^2$  reflects the reciprocal of the wage-rental ratio.

Given  $q$  and  $\bar{q}$  above, and  $(z, s)$ , the foreign and domestic firms choose  $(k, \bar{k})$  to maximize profits. For the foreign firm this yields:

$$\bar{g}_{\bar{k}} = [\bar{q} - \bar{m}\bar{k}][\bar{\beta} + (\bar{w}/2\bar{k}^2)(\bar{m}\bar{k} + \bar{q})] - \bar{q}(\partial q/\partial \bar{k}) = 0. \quad (3.22)$$

If  $\partial q/\partial \bar{k} = 0$ , as in the full precommitment case, this implies  $\bar{m}\bar{k} = \bar{q}$ , i.e., cost minimization. For the S-B case, from (3.21),

$$\partial q/\partial \bar{k} = -[\bar{w}\bar{q}/\bar{k}^2 + \bar{\beta}\bar{m}]/A < 0. \quad (3.23)$$

For the S-B case the home government chooses  $(s, z)$ , and hence  $(q, k)$ , to maximize domestic welfare,  $\pi - zq - sk$ , subject to (3.23), (3.18) and (3.19). If  $w = 0$ , so  $A$  is independent of  $k$ , solving (3.18), (3.19) and (3.22) simultaneously yields  $\bar{q}$  as a function of  $q$  alone. Thus, cost minimization for the home firm is optimal, as S-B claim. However, for  $w > 0$ ,  $\bar{q} = \Gamma(q, k)$  with  $\partial \bar{q}/\partial k > 0$ ; and it is optimal for the home firm to underinvest, as implied by Proposition 1 ( $c_{qqk} < 0$ ).

The optimal full precommitment solution is given by:

$$\begin{aligned} mk^P &= q^P = [2\alpha - \bar{\alpha}]/2, \\ \bar{m}\bar{k}^P &= \bar{q}^P = [3\bar{\alpha} - 2\alpha]/4. \end{aligned} \quad (3.24)$$

Since this case entails cost minimization for both firms, only LRMC matters. Since no analytical solution is possible for either the S-B case or the time consistent case, we rely on simulation results. In the following, we assume  $w = 0$ , so that home cost minimization is optimal under the S-B solution; we also assume  $\bar{\beta} = 0$ . The S-B solution is implicitly given by:

$$(\alpha - \bar{q} - 2q) - q(d\bar{q}/dq) = 0, \quad mk^* = q^*, \quad (3.25)$$

where  $d\bar{q}/dq$  is the slope of the foreign firm's long-run reaction function (derived from (3.18), (3.19) and (3.22)).

Finally, the time consistent solution is derived by finding  $z(k, \bar{k})$ , and substituting into (3.18) and (3.19). Using (3.5):

$$\begin{aligned} z^t &= q^t/a_{22}, \\ q^t &= (a_{22}b_0 - b_1)/(A - 1), \\ q^t &= (b_1A/a_{22} - b_0)/(A - 1). \end{aligned} \quad (3.26)$$

An increase in  $\bar{k}$  has a potentially ambiguous impact on  $z^t$  since increases in  $\bar{k}$  make the foreign reaction function more elastic, and thus (for given  $q$ ) increase the optimal subsidy. Differentiating:

$$\partial z^t/\partial \bar{k} = [(\bar{w}/\bar{k}^2)((A + 1)a_{22}b_0 - 2Ab_1)]/[a_{22}^2(A - 1)^2], \quad (3.27)$$

and

$$\partial q^t/\partial \bar{k} = [(\bar{w}/\bar{k}^2)(2b_0 - b_1a_{11})]/(A - 1)^2. \quad (3.28)$$

From (3.27), since  $a_{22} > 2$ , a sufficient condition for  $\partial z^t/\partial \bar{k} > 0$  is  $b_0 > b_1$ . Similarly, from (3.28) it is apparent that  $\partial q^t/\partial \bar{k}$  may be positive. These results are most likely to hold when the domestic firm is more efficient (i.e.,  $\alpha > \bar{\alpha}$ ).

The foreign firm's choice of  $\bar{k}$  is given by (3.22), except that the term  $\partial q/\partial \bar{k}$  is given by (3.28), rather than (3.23). If  $\partial q/\partial \bar{k} > 0$ , the foreign firm will underinvest in capital:

$$\bar{g}_{\bar{k}}^t = [\bar{q}^2 - (\bar{m}\bar{k})^2 - 2\bar{q}(2b_0 - b_1a_{11})/(A - 1)^2][\bar{w}/2\bar{k}^2] = 0. \quad (3.29)$$

Equations (3.26) and (3.29) determine  $(q, \bar{q}, \bar{k})$  as functions of  $k$ . Given these, the



home government chooses  $k(s)$  to maximize net welfare. The FOC is given by:

$$\beta m(q - mk) - (q\bar{q}/a_{22})(\bar{w}/\bar{k}^2)(d\bar{k}/dk) = 0, \quad (3.30)$$

where  $d\bar{k}/dk$  the slope of the foreign reaction function in input space. As is clear, it will be optimal to “overinvest” in capital ( $mk > q$ ) as long as this reaction function is negatively sloped.

The optimal investment subsidy/tax needed to support the solution is found from (3.15) for the time consistent solution and from (2.24) for the S-B solution. The latter must be a tax since cost minimization is optimal.

The simulations were performed as follows. Setting  $w = \bar{\beta} = 0$  for the S-B case, equations (3.18), (3.19) and (3.22) yield a cubic equation which determines foreign output and investment ( $\bar{q}, \bar{k}$ ) as functions of domestic output. For any set of parameters, this equation was solved and the derivatives ( $d\bar{q}/dq$ ) calculated. Finally, the optimal solution was found by searching over  $q$  until the value of  $q$  which solves (3.25) was determined. Subsequently, domestic and foreign welfare and the capital subsidy/tax needed to support this solution (as determined from (2.24)) were calculated. This process was then repeated for different values of the parameters.

The procedure for the time consistent case was analogous. Equations (3.26) and (3.29) yield a fourth degree polynomial which determines  $\bar{k}$  (and hence  $\bar{q}$ ) as functions of  $k$ . This equation was solved by numerical methods for a given  $k$ , and the derivative  $d\bar{k}^t/dk$  was calculated. Then, by iteration the optimal value of  $k$  (which solves (3.30)) was determined. Using this solution, the time consistent output levels, welfares and the domestic capital subsidy/tax needed to support the solution (as determined from (3.15)) were calculated. Again, this process was repeated for a wide range of parameter values.

Table 1 presents a limited number of simulation outcomes, which are representative of the general results. These simulations assume  $w = \bar{\beta} = 0$ ,  $m = \bar{w} = 1$ , and  $\beta = .8$ . The Table reports how changes in the parameters  $\alpha$ ,  $\bar{\alpha}$ , and  $\bar{m}$  affect the solutions. These parameters are, as intuition would suggest, the crucial ones.<sup>3</sup>

Results are reported for each of the models. The solution for the Full Precommitment (P-C) model depends only upon  $\alpha$ , and  $\bar{\alpha}$ , whereas the solutions for both the Spencer-Brander (S-B) model and the Time Consistent (T-C) model depends upon  $\bar{m}^2$  as well. The columns labelled (1)-(4) report welfare and output levels for the home and the foreign countries, and are self-explanatory. Columns (5) and (6) report the ratio of actual investment to the cost-minimizing level of investment (for the given output level) for the home and the foreign firms, and hence are a measure of excess capacity. Column (7) shows the domestic investment subsidy/tax needed to support the optimal solution.

The simulation results support the earlier theoretical analysis in that domestic welfare is always lower under the S-B solution than under the P-C solution, and (because of the cost specification) an investment tax is needed to support the S-B solution. Also note that domestic (and foreign) output may be higher or lower under the S-B solution as compared to the P-C solution.

The most interesting results involve comparisons of the T-C solution to either of the other solutions. Regardless of  $\alpha$  and  $\bar{\alpha}$ , domestic welfare under the T-C solution exceeds that under the S-B solution for small  $\bar{m}^2$ , although the differences are most

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<sup>3</sup>Setting  $m = 1$  is a normalization rule that affects the measurement of capital, but not the results. The key parameter in the foreign cost function is  $\bar{w}\bar{m}$ : Setting  $\bar{w} = 1$  also merely defines units of capital. The parameter  $\beta$  in the domestic cost function does affect quantitative results somewhat, but does not affect qualitative results.

Table 3.1: Comparing Alternative Solutions

Parameters			Solution Concept <sup>a</sup>	Welfare levels		Output levels		[Actual/Cost Minimum] Investment levels		Domestic Capital Subsidy
$\alpha$	$\bar{\alpha}$	$\bar{m}^2$		Home (1)	Foreign (2)	Home (3)	Foreign (4)	Home (5)	Foreign (6)	(7)
40	30		P-C Model	312.50	6.25	25.00	2.50	1.00	1.00	0.00
		0.2	S-B Model	311.64	6.29	24.98	2.55	1.00	1.18	-3.98
		0.2	T-C Model	347.21	6.26	23.07	1.71	1.14	0.11	-1.25
		2.0	S-B Model	310.11	6.23	24.99	2.60	1.00	1.16	-3.38
		2.0	T-C Model	338.69	7.78	23.19	2.01	1.15	0.34	-1.29
		20.0	S-B Model	307.07	5.51	25.26	2.58	1.00	1.11	-2.25
		20.0	T-C Model	323.00	12.05	22.72	2.85	1.15	0.74	-1.16
		200.0	S-B Model	303.83	3.39	26.26	2.17	1.00	1.04	-0.95
		200.0	T-C Model	298.51	18.47	21.40	4.38	1.18	1.01	0.41
40	35		P-C Model	253.13	39.06	22.50	6.25	1.00	1.00	0.00
		0.2	S-B Model	252.32	39.23	22.47	6.30	1.00	1.19	-3.77
		0.2	T-C Model	268.34	18.92	25.02	3.65	1.25	0.14	-1.57
		2.0	S-B Model	250.71	39.51	22.41	6.40	1.00	1.18	-3.50
		2.0	T-C Model	260.18	30.23	23.71	5.07	1.16	0.55	-4.30
		20.0	S-B Model	246.63	39.83	22.32	6.63	1.00	1.15	-2.86
		20.0	T-C Model	254.84	43.14	21.84	6.36	1.12	0.91	-3.99
		200.0	S-B Model	239.09	38.98	22.38	6.94	1.00	1.10	-1.83
		200.0	T-C Model	235.19	55.81	19.97	8.06	1.16	1.08	-1.62

40	40	P-C Model	200.00	100.00	20.00	10.00	1.00	1.00	0.00
	0.2	S-B Model	199.27	100.29	19.96	10.05	1.00	1.19	-3.40
	0.2	T-C Model	199.93	100.35	19.97	10.02	1.01	1.01	-8.40
	2.0	S-B Model	197.79	100.83	19.89	10.16	1.00	1.19	-3.23
	2.0	T-C Model	199.34	101.55	19.84	10.10	1.03	1.04	-7.50
	20.0	S-B Model	193.74	102.08	19.73	10.45	1.00	1.16	-2.80
	20.0	T-C Model	195.38	106.82	19.31	10.54	1.07	1.09	-5.64
	200.0	S-B Model	185.02	103.59	19.49	11.02	1.00	1.12	-2.00
	200.0	T-C Model	178.60	120.57	17.79	12.06	1.12	1.16	-2.94
40	45	P-C Model	153.13	189.06	17.50	13.75	1.00	1.00	0.00
	0.2	S-B Model	152.49	189.46	17.46	13.80	1.00	1.19	-2.99
	0.2	T-C Model	152.55	189.92	17.43	13.82	1.01	1.17	-7.51
	2.0	S-B Model	151.17	190.27	17.39	13.92	1.00	1.19	-2.87
	2.0	T-C Model	151.21	191.84	17.28	13.97	1.02	1.18	-7.00
	20.0	S-B Model	147.43	192.38	17.19	14.23	1.00	1.17	-2.57
	20.0	T-C Model	146.49	197.81	16.79	14.47	1.05	1.20	-5.73
	200.0	S-B Model	138.74	196.43	16.80	14.94	1.00	1.14	-1.94
	200.0	T-C Model	130.79	213.21	15.40	16.05	1.10	1.21	-3.47

<sup>a</sup> The P-C, S-B, T-C models represent the solutions to the Full Precommitment, Spencer-Brander, and Time Consistent models respectively. This simulation assumes  $\beta = 0.8$ .

pronounced for  $\alpha > \bar{\alpha}$ . Also, for  $\alpha > \bar{\alpha}$ , the T-C solution dominates the P-C solution for small  $\bar{m}^2$ , whereas for  $\bar{\alpha} \geq \alpha$ , the P-C solution is superior. Similarly, (home and foreign) output levels may be larger or smaller than output levels under either of the other two solutions, depending upon parametric values.

Turning to the investment decision, for the home firm cost-minimization prevails under both the P-C and S-B solutions, whereas under the T-C solution it is always optimal to “overinvest.” Note, however, that the foreign investment level depends upon the parameters. In particular, for  $\alpha > \bar{\alpha}$ , the foreign firm will underinvest if  $\bar{m}^2$  is not “too” large. Finally, note that despite the fact it is optimal for the home firm to overinvest, in most cases the optimal supporting policy is an investment tax, not a subsidy (though, as the Table shows, a subsidy is optimal in some cases).

The role of  $\alpha$  and  $\bar{\alpha}$  is readily interpretable in terms of equation (3.27). For  $\alpha > \bar{\alpha}$ , it is likely that  $\partial z^t / \partial \bar{k} > 0$ , and possible that  $\partial q^t / \partial \bar{k} > 0$ , which imply that the time consistent subsidy tends to discourage foreign investment (thereby raising domestic welfare). This is supported by the fact that foreign excess capacity is lower under the T-C solution than under either the S-B solution or the P-C solution for most such cases.

The role of  $\bar{m}^2$  may be less transparent. Low values of  $\bar{m}^2$  imply higher investment levels for any given  $\bar{q}$  (i.e., corresponds to capital being relatively inexpensive). However, increases in  $\bar{k}$  make the foreign *ex post* reaction function more elastic, and hence make it likely that the optimal domestic export subsidy (and domestic output levels) are increasing functions of foreign investment. Thus, for these cases the T-C solution is likely to be superior. Note, in particular, that for small  $\bar{m}^2$  (and  $\alpha > \bar{\alpha}$ ) there is significant underinvestment by foreign firms.

Hence the simulation results are in accord with the theoretical conjectures outlined earlier.

### 3.5 Concluding Remarks

We have shown that the S-B conclusion that optimal intervention by the home government entails cost minimization is incorrect. We have also shown that their solution is inferior to one in which the home government can, through appropriate policy instruments, precommit both output and investment.

We have also shown that neither of these solutions is time consistent, and have argued that, as long as governments are not permanently empowered, the time consistent solution is more likely to prevail. Surprisingly, we have shown that the time consistent solution may be superior to either precommitment solution, a result that is at odds with the conventional wisdom concerning the inferiority of the time consistent solution. This result emerges because neither solution is made optimally contingent on foreign investment (which would, of necessity, be a time inconsistent solution).

However, from a policy perspective, several major caveats are appropriate. First, the models assume the government had access to the same (complete) information set as the private sector and that it acts in a way that reflects the national interest, both highly questionable assumptions. Secondly, we ignore retaliation by the foreign government. Finally, note that although an optimal intervention policy does (by definition) exist, the direction of that policy is uncertain. The optimal policy needed to support (the S-B and the T-C) solution may be either an investment subsidy or a tax, so that without detailed information on cost (and demand) structures, the policy-maker does not know the sign, let alone the magnitude of the optimal policy.

These reservations should be sufficient to shed doubt on the possibility of an effective strategic trade policy.

## 4. PROPORTIONAL QUOTA AND COURNOT COMPETITION

### 4.1 Introduction

Many authors have examined the effects of quantity restrictions in oligopolistic international trade in terms of volume quotas and Stackelberg leadership. In reality, however, when a government negotiates the level of a quota, often its objective is to attain a target share (a percentage) of the domestic market demand for the domestic firm. In the case of U.S. steel imports, major foreign exporting nations are allocated a certain (maximum) share of the U.S. market, with each VER (Voluntary Export Restraints; or VRA (Voluntary Restraint Agreement)) varying across product categories and from year to year (Boorstein, 1987, p. 132).<sup>1</sup> Similarly, the restrictions on Japanese automobile exports were tied to U.S. market conditions. When the Japanese government announced in 1981 a three-year system of VER on the export of automobiles to the U.S. market, the export ceiling for the second year was supposed to be raised by 16.5 % of the growth in the U.S. market (Feenstra, 1984).

The examples cited above suggest that the market share arrangement is a com-

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<sup>1</sup>Restrictions in steel are so complicated that we illustrate only a few episodes from Boorstein (1987). In 1982, the major goal of commercial policy was to limit the market share of imports to about 18 % of U.S. consumption. In 1986, the market share of import was limited upto 5 % to EC, 5.8 % to Japan, and 1.9 % to Korea, excluding specialty steel. The agreements were extended in October 1989 that the VER target of 19.1 % was set.



mon instrument used to protect a nation's domestic industry, and thus it is important to analyze the effects of proportional quotas. Mai and Hwang (1989) is the only paper that investigates the effects of ratio quotas under imperfect competition.

However, they assume that the imposition of a ratio quota on the foreign exporting firm by a domestic government transforms the game structure from one of Cournot duopoly into a structure in which the domestic firm acts as a Stackelberg leader and the foreign firm as a Stackelberg follower. Before criticizing their assumptions on the behavior pattern of the rivals,<sup>2</sup> and offering an alternative analysis, we discuss the other papers' results as well as theirs.

Mai and Hwang (MH, henceforth) show that the (non) equivalence of tariffs and quotas depends on the particular value of the conjectural variations parameter, and the target ratio of imports to domestic production, and that the price under a ratio quota is always higher than that under a volume quota.

Harris (1985) demonstrates that, under a Bertrand-Nash game, a VER imposed on the foreign firm is 'voluntary' because the imposition of the VER will raise profits of both the foreign and the domestic firm producing the substitute goods. Assuming the demand for the foreign good does not exceed the VER, the paper asserts that the domestic firm takes the position of Stackelberg leadership, the foreign firm's being a price follower. The game structure is altered due to the imposition of the VER by the domestic government because "the presence of the VER means that any increase in its own price must be met by an appropriate price increase by the foreign firm so that the VER is met" (p. 804). The major conclusions are that 1) VER facilitates price leadership by the protected domestic firms and leads to an increase in profits

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<sup>2</sup>Most papers dealing with duopoly make similar assumptions.

and prices for both the domestic and the foreign firm, 2) VER induces contraction of the domestic output, 3) VER is welfare reducing, and henceforth, 4) VER may be a highly undesirable form of protection.<sup>3</sup>

Itoh and Ono (1984)<sup>4</sup> employ a Bertrand duopoly model with heterogeneous goods to examine the nonequivalence of tariffs and quotas. Their conclusions are as follows: 1) In the presence of a VER, both producers earn greater profits by setting higher prices under home leadership than under foreign leadership, and 2) the domestic prices are higher under a quota than under the equivalent tariff, whichever producer becomes a price leader under the tariff.

We make two criticisms on the assumptions in MH; these criticisms will be readily applied to other papers. First, the assertion made by MH that the ratio quota (must) bind is not necessarily true; that is, neither the domestic nor the foreign firms are compelled to choose production levels such that the quota binds. In essence, the quota represents an inequality constraint that restricts the feasible space, but does not limit it to the ray corresponding to the binding inequality. This point will be illustrated more formally to exploit the half space solution in the next section.

The second, and more important, point is the institutional mechanism by which the government enforces the quota. If, for example, the government implements the quota by first observing domestic production levels (which cannot be subsequently altered), and then tells foreign firms how much steel may be sold in the U.S. (in the same period), then it is appropriate to conclude that the domestic firm becomes a

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<sup>3</sup>Yano (1989) employs an intertemporal model to show that VER may not be a highly undesirable form of protection.

<sup>4</sup>Itoh and Ono (1982) discuss the same topic but a different model of a homogeneous good and Stackelberg leadership under Bertrand competition to yield the nonequivalence between tariffs and quotas.

Stackelberg leader (as MH conclude). On the other hand, if the government implements the restriction after the fact, by observing actual domestic sales and imports and by, for example, imposing a large penalty (tax) on the foreign firm if its actual sales exceeded the target share allocated to it, then the proportional quota does not confer a first play advantage on the domestic firm. Furthermore, if firms possess perfect information, and if the penalty is set large enough, *ex post* the proportional quota limit will always be satisfied. Hence, the nature of the game-theoretic solution depends not on the proportional quota, but rather on the way it is enforced.

In reality, the government cannot –and does not– observe domestic production before deciding on the permissible level of imports. For example, under the steel agreements, market demand is forecast for the coming period, and then foreign firm import levels are assigned (as percents of this forecast). Given this forecast, domestic and foreign firms act simultaneously. Furthermore, if actual imports exceed the target share, a “penalty” is imposed by adjusting permissible imports for the subsequent period. Under this implementation, which is customary, the strategic behavior of firms is most appropriately modeled as simultaneous play, not as Stackelberg leadership for the home firm.

In fact, assuming that two firms play simultaneously before and after the imposition of a quota on the foreign firm, Krishna (1985, 1989) analyzes the effects of a volume quota under Bertrand–Nash equilibrium. The effects of the restriction depends on whether imports are substitutes or complements.<sup>5</sup> In the former case,

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<sup>5</sup>Krishna (1985, 1989) is different from Harris (1985), and Itoh and Ono (1984) in that the latter two papers consider only substitute goods and Stackelberg leadership. Harris is different from Itoh and Ono in that they consider Stackelberg leadership of the domestic and the foreign firm, while Harris considers Stackelberg leadership of only the domestic firm.

no equilibrium exists in pure strategies, but there exists the unique mixed strategy equilibrium, where the domestic firm randomizes its production, while the foreign firm does not. As a result, the VER facilitates collusion to raise prices and profits of both firms. The paper also shows that 1) tariffs and quotas are nonequivalent if goods are substitutes, 2) the profits of the domestic firm are higher under the VER than under the equivalent tariff, 3) the VER is preferred by the foreign firm to no restriction, and 4) the foreign firm would prefer the VER to the equivalent tariff, even if the tariff revenues were returned to it as a lump sum.

In Section 2, we examine the effects of a ratio quota when the domestic firm takes Stackelberg leadership due to the imposition of a VER, where the VER need not bind. We show that there is some unique value of the quota such that the domestic firm's profits associated with MH are equal to, larger or smaller than the profits associated with the position of a normal Stackelberg leader.

We assume that the two firms play simultaneously under Cournot competition in Section 3. We show that there is no equilibrium in pure strategies, but that there exists a unique mixed strategy equilibrium such that the domestic firm randomizes its production, the high output with the probability  $\rho$ , and the low with  $1 - \rho$ , if the foreign firm chooses output to maximize its expected profit. The profits of the domestic firm increase, while those of the foreign firm may increase or decrease.

In Section 4, the effects of a VER are compared with those of an import equivalent tariff.

Concluding remarks follow.

## 4.2 Solutions for Stackelberg Leadership

MH, employing a conjectural variations model of Cournot duopoly, examine the effects of a proportional VER to compare it with the equivalent tariff and a volume quota. They claim that “it warrants mention that ..., but with a ratio quota the foreign firm is subject to the ratio constraint. Hence, the foreign firm’s reaction function becomes  $q_2 = rq_1$ ,”<sup>6</sup> and that “As a result, the domestic firm acts as a Stackelberg leader and the foreign firm as a follower.”

If the enactment of a ratio quota actually confers first-play advantage on the domestic firm (e.g., if the government must first observe domestic output before any imports enter the country), then the domestic firm is not compelled to choose an output level that will induce the follower (the foreign firm) to fulfill its quota. Thus, given the output choice of the domestic firm (denoted by  $y$ ), and the ratio quota, denoted by  $\theta$ ,<sup>7</sup> the foreign firm chooses its production level ( $x_p$ ) and its exports or sales ( $x$ ), such that  $x \leq \min(\theta y, x_p)$ .<sup>8</sup>

The foreign firm maximizes its profit ( $\bar{\pi}$ ),<sup>9</sup> given  $y$ , over  $x_p$  subject to its constraint as follows:

$$\bar{\pi} = xp(x + y) - \bar{c}(x_p), \quad (4.1)$$

subject to  $x \leq \min(x_p, \theta y)$ , and  $\bar{c}(x_p)$  denotes the total cost function of the foreign

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<sup>6</sup> $q_2$  ( $q_1$ ) is the sale of the foreign (domestic) firm and  $r$  is a ratio of a quota.

<sup>7</sup>We define  $\theta$  as the maximum ratio of imports to domestic production:  $x \leq \theta y$ . The market share of imports,  $\frac{x}{x+y}$ , is  $\frac{\theta}{1+\theta}$ . Obviously, there is a one-to-one correspondence between  $\theta$  and this import share.

<sup>8</sup>For the Stackelberg model, the foreign firm chooses  $x$  and  $x_p$  simultaneously, so the distinction is unnecessary. However, we introduce this distinction here because it is crucial to understanding the simultaneous play, and mixed strategy solution.

<sup>9</sup>The barred variables denote the foreign firm unless otherwise stated.

firm.

Assuming marginal revenue is always positive ( $p + x_p p' > 0$ ), thus clearly  $x = x_p$ , the choice of  $x_p$  from the profit function of the foreign firm is:

$$\frac{\partial \bar{\pi}}{\partial x_p} = p + x_p p' - \bar{c}', \quad (4.2)$$

where  $p + x_p p'$  is marginal revenue, and  $\bar{c}'$  marginal cost of the foreign firm.

If  $\frac{\partial \bar{\pi}}{\partial x_p} |_{x_p=\theta y} > 0$ , then  $x_p = \theta y$ ; that is, the quota is binding. If  $\frac{\partial \bar{\pi}}{\partial x_p} |_{x_p=\theta y} < 0$ , then  $x_p = \sigma(y)$ , where  $\sigma$  is the reaction function of the foreign firm. We assume that the slope of the reaction function ( $\frac{dx_p}{dy} = \sigma'(y)$ ) is negative such that  $-1 < \sigma'(y) < 0$ .<sup>10</sup>

Define  $\tilde{y}(\theta)$ , given  $\theta$ , such that  $\frac{\partial \bar{\pi}}{\partial x_p} |_{x_p=\theta \tilde{y}} = 0$ , where  $\frac{d\tilde{y}}{d\theta} < 0$ .<sup>11</sup>

Thus, the optimal solution is:  $x_p = \theta y$  if  $y \leq \tilde{y}(\theta)$ , and  $x_p = \sigma(y)$  if  $y \geq \tilde{y}(\theta)$ .

Figure 1 illustrates the kinked reaction function (with a solid line) of the foreign firm due to the constraint.  $FF'$  ( $DD'$ ) is the reaction function of the foreign (domestic) firm when there are no restrictions.<sup>12</sup> The ray  $OC$  represents a ratio quota restriction. The ray does not pass through the intersection of  $FF'$  and  $DD'$  to reflect the fact that a ratio quota ( $\theta$ ) need not be set at the free trade levels.

<sup>10</sup>It is a usual assumption in this area.

<sup>11</sup>To see  $\frac{d\tilde{y}}{d\theta} < 0$ , we take the total differentiation of  $\frac{\partial \bar{\pi}}{\partial x_p}$  at  $x_p = \theta y$  such that  $\frac{dy}{d\theta} = -y \left( \frac{\partial^2 \bar{\pi}}{\partial x_p^2} \right) / \left( \theta \frac{\partial^2 \bar{\pi}}{\partial x_p^2} + \frac{\partial^2 \bar{\pi}}{\partial x_p \partial y} \right)$ , where its own second partial is more negative than its cross partial is by a usual assumption in this area.

<sup>12</sup>The intersection between  $FF'$  and  $DD'$  represents Cournot-Nash equilibrium. The outputs of the foreign and the domestic firm associated with the equilibrium are the levels of free trade. The figure depicts the case in which  $\theta$  is less than the levels prevailing under unrestricted trade ( $\theta < x^c/y^c$ ), where  $(x^c, y^c)$  is the unrestricted Cournot solution. However, this need not be the case. i.e., even for  $\theta > x^c/y^c$ , the quota and the first play assumption change the solution.

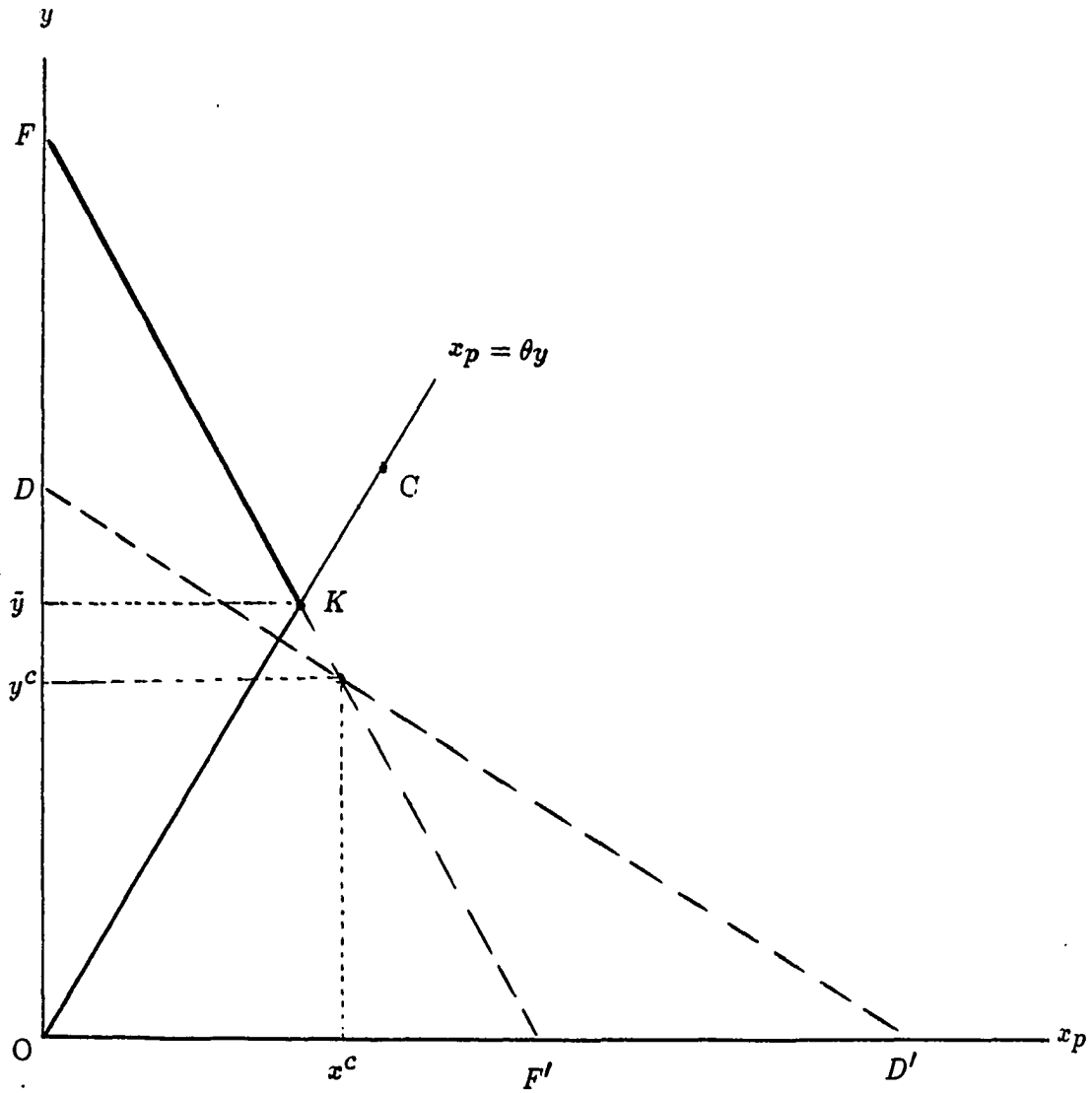


Figure 4.1: The Kinked Reaction Function of the Foreign Firm

As a Stackelberg leader (by assumption), the domestic firm utilizes the reaction function of the foreign firm to derive its optimal choice of  $y$ . The domestic firm's profit function is:  $\pi = yp(x + y) - c(y)$ , where  $c(y)$  is the total cost. We define the profit function as:

$$\pi = \begin{cases} \pi^m(y, \theta) = yp(\theta y + y) - c(y) & \text{for } y \leq \bar{y}(\theta), \\ \pi^l(y) = yp(\sigma(y) + y) - c(y) & \text{for } y \geq \bar{y}(\theta) \end{cases}$$

Further, define  $y^m(\theta)$  as the value of  $y$  that maximizes  $\pi^m$ :  $\pi_y^m \equiv \frac{\partial \pi^m}{\partial y} = p + y(1 + \theta)p' - c' = 0$  at  $y = y^m(\theta)$ . Note that  $\frac{dy^m}{d\theta} < 0$ , given the SOC, since  $\frac{\partial^2 \pi^m}{\partial \theta \partial y} = 2yp' + (1 + \theta)y^2 p'' < 0$ .<sup>13</sup>

Also, define  $y^l$  as the value of  $y$  that maximizes  $\pi^l$ :  $\pi_y^l \equiv \frac{\partial \pi^l}{\partial y} = p + y(1 + \sigma')p' - c' = 0$  at  $y = y^l$ . Note that  $y^l$  is independent of  $\theta$ .

Finally, define  $y^*$  as the value of  $y$  that maximizes domestic profits. Since  $p' < 0$ , and  $\theta > 0 > \sigma'$ , it is obvious that  $\pi_y^m < \pi_y^l$  for all  $y$ . Hence:

*Proposition 1:*

If  $y^m(\theta) \geq \bar{y}(\theta)$  (i.e.,  $\frac{\partial \pi^m}{\partial y}|_{y=\bar{y}(\theta)} \geq 0$ ), then  $y^* = y^l \geq \bar{y}(\theta)$ .

If  $y^l \leq \bar{y}(\theta)$  (i.e.,  $\frac{\partial \pi^l}{\partial y}|_{y=\bar{y}(\theta)} \leq 0$ ), then  $y^* = y^m(\theta) \leq y^l$ .

*Proof:*

The proof is immediate, given  $\pi_y^l > \pi_y^m$ . Thus,

$$\frac{\partial \pi}{\partial y} = \begin{cases} \frac{\partial \pi^m}{\partial y} & \text{for } y < \bar{y}(\theta), \\ \frac{\partial \pi^l}{\partial y} & \text{for } y > \bar{y}(\theta) \end{cases}$$

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<sup>13</sup>We assume that the demand function is not too convex.



Thus,  $\pi$  is continuous, but not everywhere differentiable in  $y$ . Given  $\pi^m$  and  $\pi^l$  as concave in  $y$ , if  $\pi_y^m \geq 0$  at  $y = \bar{y}(\theta)$ , then  $\frac{\partial \pi}{\partial y} > 0$  for  $y < \bar{y}(\theta)$  and  $\pi$  is locally increasing in  $y$  for  $y > \bar{y}(\theta)$  in the neighborhood of  $\bar{y}(\theta)$ ; hence, the maximum occurs at  $y^* = y^l > \bar{y}(\theta)$ . Similarly, for  $\pi_y^l \leq 0$  at  $y = \bar{y}(\theta)$ ,  $\pi$  is decreasing in  $y$  for all  $y > \bar{y}(\theta)$  and locally decreasing in  $y$  for  $y < \bar{y}(\theta)$  (but in the neighborhood of  $\bar{y}(\theta)$ ). Thus, the maximum occurs at  $y^* = y^m(\theta) < \bar{y}(\theta)$ . Furthermore, note that if  $y^* = y^l > \bar{y}(\theta)$ ,  $x = \sigma(y^*) < \theta y$ , so that the constraint on foreign imports is nonbinding, whereas if  $y^* = y^m(\theta)$ ,  $x = \theta y$  and the constraint is binding. *Q. E. D.*

Since  $\pi$  is not necessarily globally concave in  $y$  (given  $\theta$ ), two local optima can occur. In particular, if  $\frac{\partial \pi^m}{\partial y}|_{y=\bar{y}(\theta)} < 0 < \frac{\partial \pi^l}{\partial y}|_{y=\bar{y}(\theta)}$ , then  $y^m(\theta) < \bar{y}(\theta) < y^l$ , and the optimum optimum cannot be ascertained solely from the FOC; a comparison of the two local optima is required.

Specifically, define  $\theta^l = x^l/y^l$ , i.e.,  $\theta^l$  is the ratio of imports to domestic production under an unrestricted Stackelberg solution. Then, it is immediately apparent that:  $\frac{\partial \pi^l}{\partial y}|_{y=\bar{y}(\theta)} \gtrless 0$ , as  $\theta \gtrless \theta^l$ . Hence for  $\theta < \theta^l$ , the optimal solution is  $y^* = y^m(\theta)$ .

Similarly, define  $\theta^c = x^c/y^c$ , i.e.,  $\theta^c$  is the ratio of imports to domestic production under an unrestricted Cournot solution. Then:  $\frac{\partial \pi^m}{\partial y}|_{y=y^c} = p + (1 + \theta^c)y^c p' - c'(y^c) = \theta^c y^c p' < 0$ , where  $y^c = \bar{y}(\theta^c)$ . Hence, there is a range of  $\theta$  for which  $\pi$  has multiple local optima.<sup>14</sup>

<sup>14</sup>Note  $\bar{y}(\theta)$  solves  $p[(1 + \theta)\bar{y}] + \theta \bar{y} p'[(1 + \theta)\bar{y}] - \bar{c}'(\theta \bar{y}) = 0$ , and  $\frac{\partial \pi^m}{\partial y}|_{y=\bar{y}(\theta)} = p[(1 + \theta)\bar{y}] + (1 + \theta)\bar{y} p'[(1 + \theta)\bar{y}] - c'(\bar{y}) = \bar{y} p' + \bar{c}'(\theta \bar{y}) - c'(\bar{y})$ . Clearly, if  $c'(\bar{y}) \geq \bar{c}'(\theta \bar{y})$  for all  $\theta$  and  $\bar{y}$  as would happen for constant marginal cost, then  $\frac{\partial \pi^m}{\partial y}|_{y=\bar{y}(\theta)} < 0$

We now compare the domestic firm's profits associated with a MH solution to those associated with a normal Stackelberg solution. Since  $y^l$  is independent of  $\theta$ ,  $x^l = \sigma(y^l)$ , and  $\pi^l = \pi(x^l, y^l)$  are also independent of  $\theta$ . Since  $y^m$  depends on  $\theta$ , so do  $x^m = \theta y^m$ , and  $\pi^m = \pi(x^m(\theta), y^m(\theta))$ .<sup>15</sup> Thus,

*Lemma 1:*

If  $\theta \leq \theta^l \equiv x^l/y^l$ , then  $\pi^* = \pi^m(\theta) > \pi^l$ , and  $y^* = y^m(\theta)$ .

This result is illustrated in Figure 2, where  $S$  denotes the unrestricted Stackelberg solution, the curve  $ASB$  the domestic iso-profit curve passing through  $S$ , and  $OS\theta^l$ , the ray of  $x = \theta^l y$ . At  $\theta = \theta^l$ , the foreign reaction function is  $FSO$ ; given the constraint,  $S$  is achievable, as are points (along the ray  $OS\theta^l$ ) which are strictly preferred to  $S$  (by the home firm). Similarly, for  $\theta < \theta^l$ , points along the ray which strictly dominate the Stackelberg solution exist; hence, for  $\theta < \theta^l$ ,  $\pi^* = \pi^m(\theta) > \pi^l$ .

By continuity, for  $\theta > \theta^l$ , but near  $\theta^l$ , the optimal solution is still given by  $y^* = y^m(\theta)$ . However, as  $\theta$  increases,  $\pi^m(\theta)$  decreases. Thus:

*Proposition 2:*

There exists a  $\hat{\theta} > \theta^l$  such that  $\pi^m(\theta) \geq \pi^l$  as  $\theta \leq \hat{\theta}$ , and thus, for  $\theta < \hat{\theta}$ ,  $y^* = y^m(\theta)$ , and  $x = \theta y^m$ . For  $\theta > \hat{\theta}$ ,  $y^* = y^l$ , and  $x = x^l < \theta y^l$ .

*Proof:*

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for all  $\theta$ , so that  $y^m(\theta) < \bar{y}(\theta)$  for all  $\theta$ . Hence, for all  $\theta > \theta^l$ , multiple local solutions,  $y^m(\theta)$  and  $y^l$ , occur such that  $y^m(\theta) < \bar{y}(\theta) < y^l$ , for all  $\theta > \theta^l$ .

<sup>15</sup>It is trivial to show that  $\frac{dx^m}{d\theta} > 0$ , and  $\frac{d\pi^m}{d\theta} < 0$ .

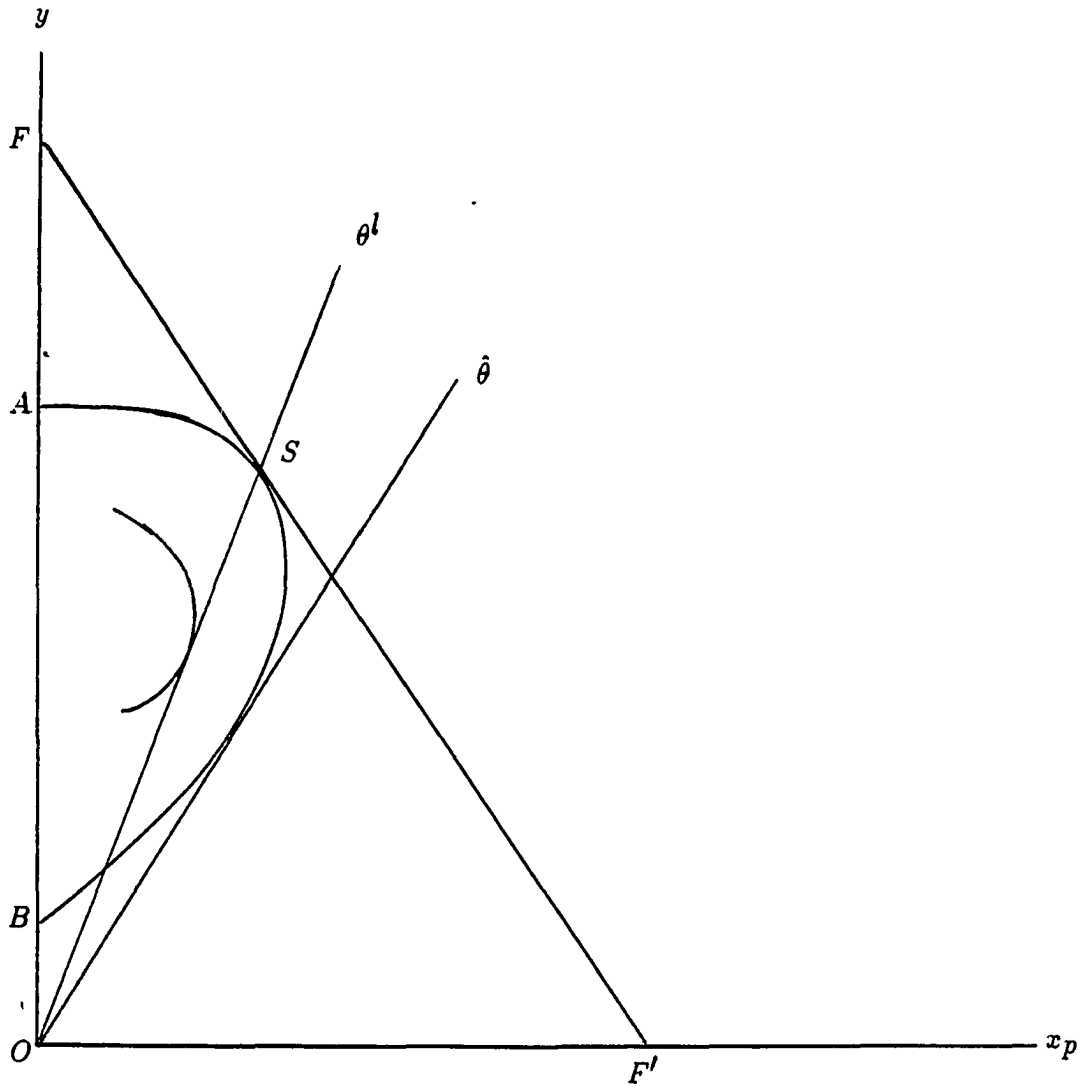


Figure 4.2: Relationship between MH and Normal Stackelberg solutions

Clearly,  $\lim_{\theta \rightarrow \infty} \pi^m(\theta) = 0 < \pi^l$ , and  $\pi^m(\theta^l) > \pi^l$ . Since  $\pi^m(\theta)$  is continuous, and decreasing in  $\theta$ , there exists  $\hat{\theta} > \theta^l$  such that  $\pi^m(\hat{\theta}) = \pi^l$ . *Q.E.D.*

Figure 2 shows that  $\hat{\theta}$  is determined by finding a ray from the origin that is tangential to the iso-profit curve  $\pi^l(x, y) = \pi^l$ . The slope of this ray is  $1/\hat{\theta}$ . Figure 3 shows the comparison of profits of MH ( $\pi^m$ ) and of a normal Stackelberg solution ( $\pi^l$ ). It is self-explanatory.

Figure 4 shows the levels of output of the home and the foreign firms as the ratio quota changes. At  $\theta = 0$ , the home firm produces monopoly output with the foreign firm's zero output. As  $\theta$  increases to  $\hat{\theta}$ ,  $y$  decreases, and  $x$  increases.<sup>16</sup> At  $\hat{\theta}$ ,  $y$  jumps to  $y^l > y^m(\hat{\theta})$  and stays there to keep the same profit level, and  $x$  jumps to  $x^l$ , which, however, may be larger or smaller than  $x(\hat{\theta}) = \hat{\theta}y^m(\hat{\theta})$ .

Note that the magnitude of  $\hat{\theta}$  in relation to the Cournot level ( $\theta^c \equiv x^c/y^c$ ) cannot be determined on *a priori* grounds since, as previously shown, there are multiple local optima for  $\pi$ .

For the special case of linear demand and constant marginal cost, results can be obtained. Let the demand function and cost functions be:  $p = A - B(x + y)$ ,  $c(y) = cy$ , and  $\bar{c}(x) = \bar{c}x$ .

Then, using the solution concepts previously defined, we have:  $y^l = (A + \bar{c} - 2c)/2B$ ,  $x^l = (A + 2c - 3\bar{c})/4B$ ,  $\pi^l = (A + \bar{c} - 2c)^2/8B$ ,  $y^m = (A - c)/[2B(1 + \theta)]$ ,  $x^m = \theta y^m$ ,  $\pi^m = (A - c)^2/[4B(1 + \theta)]$ , and  $\tilde{y}(\theta) = (A - \bar{c})/[B(1 + 2\theta)]$ .

Note that  $y^m(\theta) < \tilde{y}(\theta)$  if and only if  $[A - \bar{c} + (c - \bar{c})(1 + 2\theta)] > 0$ . Thus, for

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<sup>16</sup>The vertical axis for the variable  $x$  is such constructed that going down the axis is increasing the value of  $x$ .

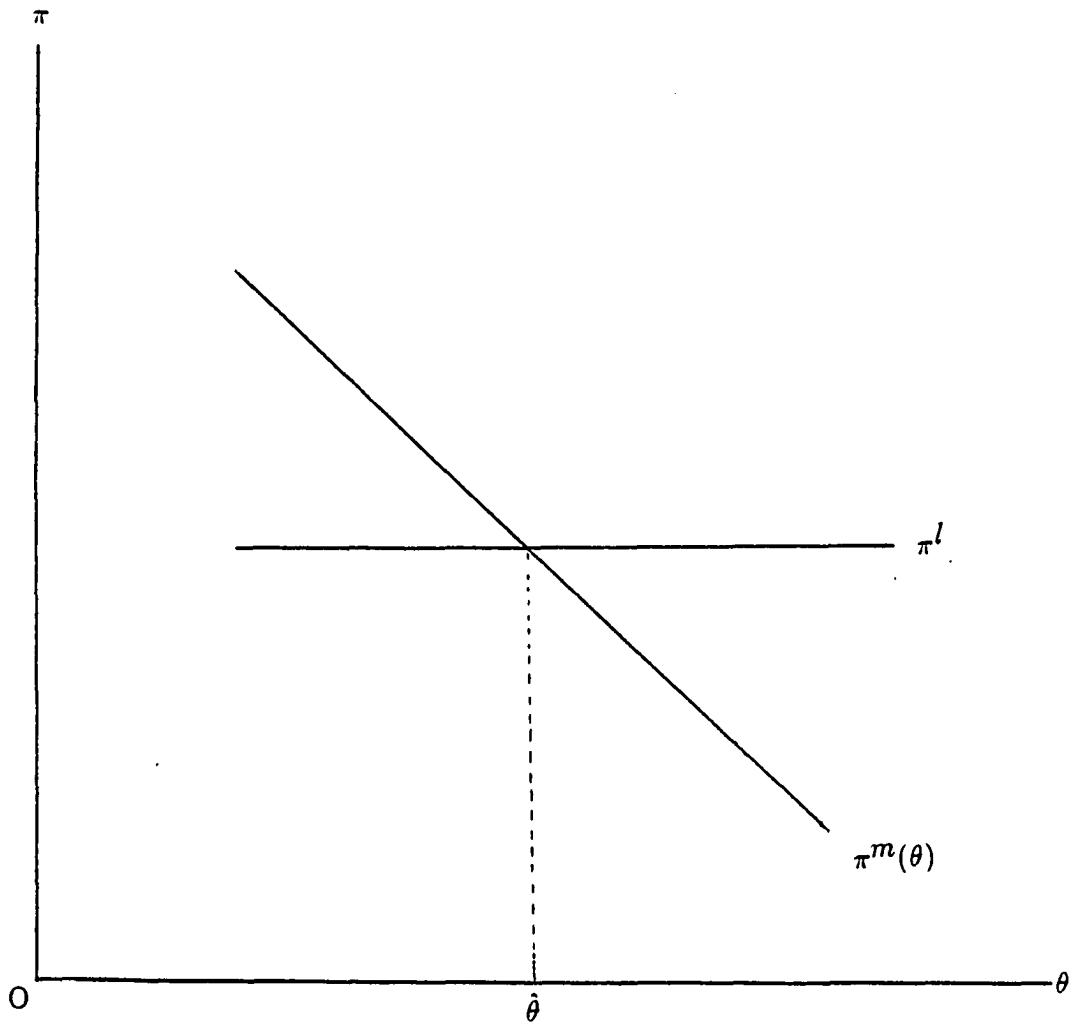
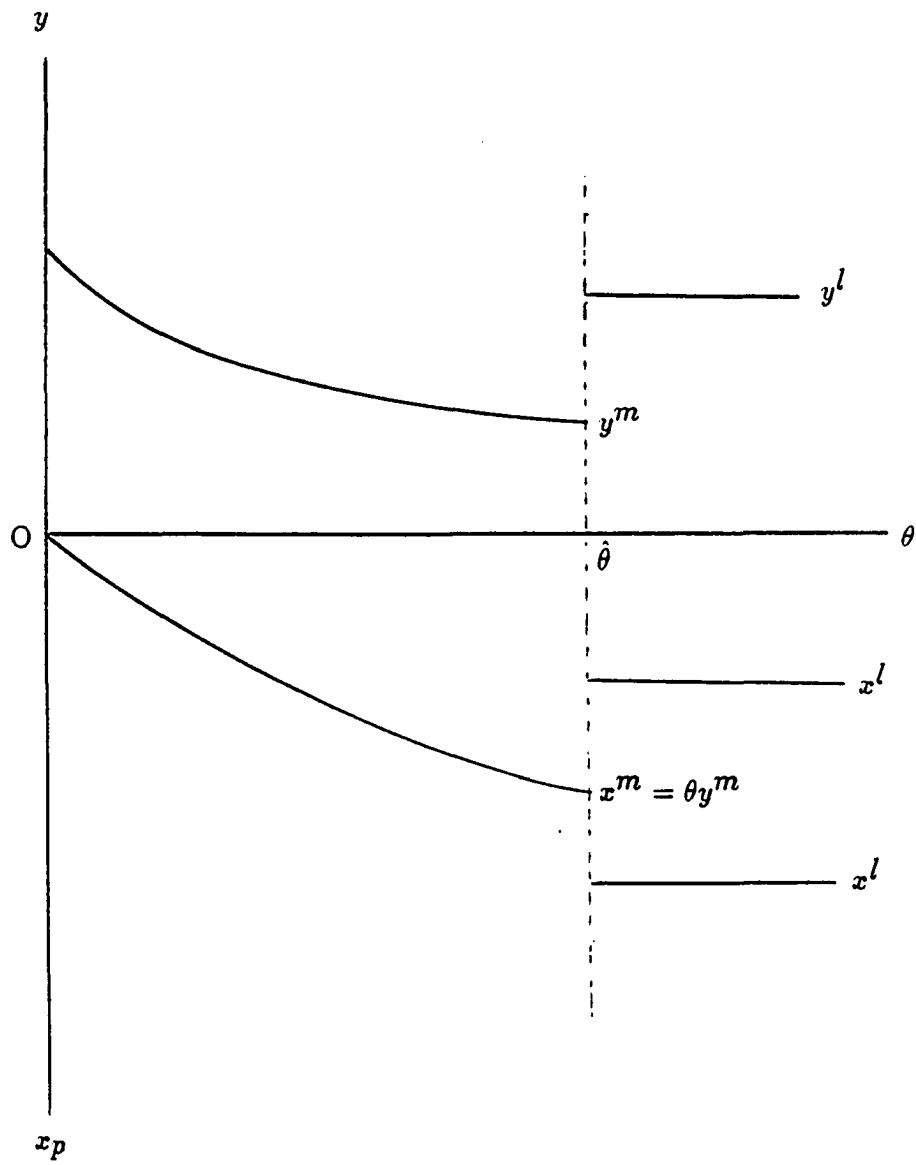


Figure 4.3: The Comparison of Profits of MH and Normal Stackelberg Solution

Figure 4.4: Output Levels and  $\theta$

$c \geq \bar{c}$ ,  $y^m(\theta) < \bar{y}(\theta)$  for all  $\theta$ .

Hence,  $\hat{\theta}$  solves  $\pi^m = \pi^l$  such that:

$$1 + \hat{\theta} = 2[(A - c)^2 / (A - 2c + \bar{c})^2]. \quad (4.3)$$

Furthermore, for the Cournot solution  $(x^c, y^c)$ , we have:  $x^c = (A + c - 2\bar{c})/3B$ ,  $y^c = (A + \bar{c} - 2c)/3B$ , and  $1 + \theta^c = (2A - (c + \bar{c})) / (A + \bar{c} - 2c)$ . Hence  $\hat{\theta} \stackrel{\geq}{<} \theta^c$  as  $c \stackrel{\geq}{<} \bar{c}$ .

*Proposition 3:*

In the case of linear demand and constant marginal cost, if a ratio quota is imposed at the unrestricted Cournot trade levels, and if the implementation of the quota gives a first play advantage to the domestic firm, then,

1) if  $\bar{c} > c$ , the solution will entail  $y^* = y^l$  and the quota will not bind. The home firm's output and profits rise, the foreign firm's output and profits fall, and total output rises. Domestic welfare unambiguously rises.

2) if  $\bar{c} < c$ , the resulting solution entails  $y^* = y^m(\theta)$ , and the quota binds; domestic profits rise; as do foreign profits; total output falls, as does welfare.

3) if  $\bar{c} = c$ , the the two local solutions are also global solutions; the resulting equilibrium is not unique.

#### 4.2.1 Summary

The impact of a ratio quota on the equilibrium trade levels depends not only on the magnitude of the quota, but also on the solution concept used to derive the equilibrium. In this section, following MH, we have analyzed the solution assuming that the implementation of the quota conveys a first play advantage on the domestic

firm. Within this framework, we have found that:

1) with the quota and first play advantage, the domestic firm's maximum profits must be at least as large as the unrestricted Stackelberg profit level,

2) there exists a critical value of the ratio quota,  $\hat{\theta}$ , such that if  $\theta > \hat{\theta}$ , the unrestricted Stackelberg solution emerges,

3) if  $\theta < \hat{\theta}$ , the quota will bind in the equilibrium, and

4) *a priori*, the critical value  $\hat{\theta}$  may be greater or less than the Cournot trade ratio.

It is important to emphasize that the results obtained in this section depend not only on the ratio quota, but also on the assumption that the presence of the quota changes the game structure. Earlier we argued that this assumption –made by MH– is generally unwarranted. The real issue is how the government implements the law, and it is our belief that the way such laws are usually implemented does not convey a first play advantage. In the next section, we study how a ratio quota affects the equilibrium, assuming simultaneous play by two firms.

### 4.3 Simultaneous Play

In this section firms are assumed to be Cournot competitors (before and) after the ratio quota is imposed. Since the pre-quota game was analyzed in the previous section, we present only the post-quota game in this section.

Two firms, one domestic and one foreign, produce goods which are perfect substitutes and are sold in the domestic country. The domestic government is assumed to be active to protect its own firm, while the foreign government does not pursue an active policy. Specifically, the domestic government passes a law which restricts



imports to be no larger than a certain fraction of the domestic market. The law is implemented as follows; actual imports and domestic sales (in a given period) are observed, and if imports exceed the target level specified in the law, then a large penalty is imposed upon the foreign exporter. Given perfect information at the time sales decisions are made, if the penalty is large enough the foreign firm will always, *ex post*, choose to be in compliance with the law. The major point is that implementation of the law does not require the domestic government to observe production and sales of the domestic firm before issuing import licenses, and hence the law does not give a first play advantage to the home firm. As we have argued earlier, it is our belief that this description of the way the law is implemented is in close conformity with actual practice.<sup>17</sup>

In order to set the stage for a mixed strategy solution, which we will show is the only consistent simultaneous play solution, we assume that decisions are made in the following sequence: i) the government sets the ratio quota,  $\theta$ ; ii) given  $\theta$ , domestic and foreign firms simultaneously choose their output levels, denoted  $(y_p, x_p)$ ; iii) given the predetermined production levels and  $\theta$ , firms choose their sales levels, denoted  $(y, x)$  subject to  $y \leq y_p$  and  $x \leq \min(x_p, \theta y)$ .

Clearly, given perfect information and assuming that firms pursue pure strategies, the separation of the timing of production and sales decisions does not matter; however, for a mixed strategy solution in which the home firm randomizes its production decisions, this separation is necessary to insure compliance with the quota.

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<sup>17</sup>Indeed, it is hard to see how the alternative version of the law, in which the government had to observe current domestic production and sales before allowing current imports, could be implemented; further, prices for domestic and import goods could differ in this setting. In a multi-period framework, this formulation makes more sense.

Assuming marginal revenue is positive everywhere, the sales decision is trivial, and the equalities will bind such that  $y = y_p$  and  $x = \min(x_p, \theta y)$ . Hence, the foreign firm –if it knows  $y_p$  (and hence  $y$ ) when it chooses  $x_p$  (as in the case of a perfect information, pure strategy model)– will behave as modeled in the previous section. Since the domestic and the foreign firm choose output levels simultaneously, let  $y^e$  denote the foreign firm's (point expectation) concerning the output decisions of the home firm. The foreign firm's profit function is given by:

$$\bar{\pi}(x_p, y^e) = x_p p(x_p + y^e) - \bar{c}(x_p), \quad (4.4)$$

which it maximizes subject to  $x_p \leq \theta y^e$ . As in the previous section, the solution is:

$$x_p = \begin{cases} \theta y^e & \text{if } y^e < \bar{y}(\theta), \\ \sigma(y^e) & \text{if } y^e > \bar{y}(\theta), \end{cases}$$

where  $\bar{y}(\theta)$  solves  $\frac{\partial \bar{\pi}}{\partial x_p} |_{x_p = \theta \bar{y}} = 0$ .

Thus, the behavior of the foreign firm depends on its beliefs ( $y^e$ ) concerning the output of the home firm, not on the actual output level (though, in a perfect foresight equilibrium, the beliefs are correct). The reaction function of the foreign firm (with respect to beliefs,  $y^e$ ) is *FKO* as shown in Figure 5.

The home firm takes the foreign firm's production decision,  $x_p$ , as given, but knows that subsequent sales ( $x$ ) cannot exceed  $\theta y$ . Hence, the home firm's profit function is given by:

$$\pi = y p[\min(x_p, \theta y) + y] - c(y), \quad (4.5)$$

where  $x = \min(x_p, \theta y)$ . For convenience, define:

$$\pi^i(y, \theta) \equiv y p[(1 + \theta)y] - c(y), \quad (4.6)$$

$$\pi^{ii}(x_p, y) \equiv y p(x_p + y) - c(y). \quad (4.7)$$

Thus, we have

$$\pi(x_p, y, \theta) = \begin{cases} \pi^i(y, \theta) & \text{for } y \leq x_p/\theta, \\ \pi^{ii}(y, x_p) & \text{for } y \geq x_p/\theta, \end{cases}$$

Furthermore,  $\pi^i(y, \theta) = \pi^{ii}(x_p, y)$  at  $y = x_p/\theta$ , so that  $\pi(x_p, y, \theta)$  is continuous in  $y$ , but not everywhere differentiable such that:

$$\frac{\partial \pi}{\partial y} \equiv \begin{cases} \pi_y^i = p[(1 + \theta)y] + (1 + \theta)yp' - c' & \text{for } y < x_p/\theta, \\ \pi_y^{ii} = p(x_p + y) + yp' - c' & \text{for } y > x_p/\theta, \end{cases}$$

where  $\pi_y^i = \frac{\partial \pi^i}{\partial y}$ ,  $\pi_y^{ii} = \frac{\partial \pi^{ii}}{\partial y}$ , and  $c' = c'(y)$ .

Assuming  $\pi^i(y, \theta)$  and  $\pi^{ii}(x_p, y)$  are each globally concave in  $y$ , define  $y^m = \mu(\theta)$  as the value of  $y$  which solves  $\pi_y^i = 0$ ;  $\mu(\theta)$  is the MH solution to the Stackelberg leader problem. Also, define  $y^p = B(x_p)$  as the value of  $y$  which solves  $\pi_y^{ii} = 0$ , which is the standard Cournot reaction function for unrestricted simultaneous play.

Note that  $\pi_y^{ii}(x_p, y) > \pi_y^i(y, \theta)$  at  $y = x_p/\theta$  since  $\theta > 0 > p'$ . Define  $y^*(x_p, \theta)$  as the value of  $y$  that maximizes  $\pi(x_p, y, \theta)$ , given  $(x_p, \theta)$ ; then we have,

If  $\pi_y^i|_{y=x_p/\theta} > 0$ , then  $y^*(x_p, \theta) = B(x_p) > x_p/\theta$ .

If  $\pi_y^{ii}|_{y=x_p/\theta} < 0$ , then  $y^*(x_p, \theta) = \mu(\theta) < x_p/\theta$ .

If  $\pi_y^i|_{y=x_p/\theta} < 0 < \pi_y^{ii}|_{y=x_p/\theta}$ , then there are two local solutions  $y_1$  and  $y_2$  such that  $y_1 = y^p = B(x_p) > x_p/\theta$  and  $y_2 = y^m = \mu(\theta) < x_p/\theta$ ; the global optimum cannot be inferred from the FOC.

Define  $\dot{x}(\theta) = \theta\mu(\theta)$ . Then, by definition of  $\mu(\theta)$ , if  $x_p \leq \dot{x}(\theta)$ , then  $\pi_y^i \geq 0$  for all  $y \leq x_p/\theta$ , and hence the optimal solution is given by  $y^* = B(x_p) > x_p/\theta$ .

Similarly, define  $\ddot{x}(\theta) (> \dot{x}(\theta))$  such that  $\pi_y^{ii} = 0$  at  $x_p = \ddot{x}(\theta)$  and  $y = \ddot{x}/\theta$ ; thus,  $\ddot{x}(\theta)$  solves  $B(x) = \theta x$ . Then, for all  $x_p \geq \ddot{x}(\theta)$ ,  $\pi_y^{ii} \leq 0$  for  $y \geq x_p/\theta$ , implying the

optimal solution is  $y^*(x_p, \theta) = \mu(\theta)$  in this domain. These solutions are illustrated in Figure 5, where  $DA$  corresponds to the solution  $B(x_p)$  and the horizontal line  $(ab)$  corresponds to the MH solution  $\mu(\theta)$ .

Finally, for  $x_p \in (\dot{x}(\theta), \ddot{x}(\theta))$ , there are two local solutions. Define:

$$\pi^m(\theta) = \pi^i(y, \theta), \quad (4.8)$$

where  $y = \mu(\theta)$ , and

$$\pi^P(x_p) = \pi^{ii}(x_p, y), \quad (4.9)$$

where  $y = B(x_p)$ .

For  $x_p = \dot{x}(\theta)$ ,  $B(\dot{x}) > \dot{x}/\theta = \mu(\theta)$ , and thus;

$$\pi^P(\dot{x}) > \pi^{ii}(\dot{x}, y)|_{y=\dot{x}/\theta} = \pi^i(y, \theta)|_{y=\dot{x}/\theta} = \pi^m(\theta). \quad (4.10)$$

Hence, global optimum occurs at  $y^* = B(\dot{x})$ .

Similarly, at  $x_p = \ddot{x}(\theta)$ ,  $B(\ddot{x}) = \ddot{x}(\theta)/\theta > \mu(\theta)$ , and ;

$$\pi^P(\ddot{x}) = \pi^{ii}(\ddot{x}, y)|_{y=\ddot{x}/\theta} = \pi^i(y, \theta)|_{y=\ddot{x}/\theta} < \pi^m(\theta). \quad (4.11)$$

Hence, the global optimum occurs at  $y^* = \mu(\theta)$ . Since  $\pi^m(\theta)$  is independent of  $x_p$ , by continuity there exists  $\hat{x} \in (\dot{x}(\theta), \ddot{x}(\theta))$  such that  $\pi^m(\theta) = \pi^P(\hat{x}(\theta))$ . Since  $\pi^m(\theta)$  is a decreasing function of  $\theta$ , and  $\pi^P(x_p)$  is a decreasing function of  $x_p$ ,  $\hat{x}(\theta)$  must be an increasing function of  $\theta$ .<sup>18</sup>

Thus, given  $x_p$ , the domestic firm's optimal solution is characterized as follows:

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<sup>18</sup>  $\hat{x}(\theta)$  solves  $\pi^m(\theta) = \pi^P(\hat{x}(\theta))$ , where  $\frac{\partial \pi^m}{\partial \theta} = \mu^2(\theta)p'((1 + \theta)\mu(\theta)) < 0$ , and  $\frac{\partial \pi^P}{\partial x_p} = yp'(x + y) < 0$ . Thus,  $\frac{d\hat{x}}{d\theta} = \frac{\partial \pi^m}{\partial \theta} / \frac{\partial \pi^P}{\partial x_p} > 0$ .

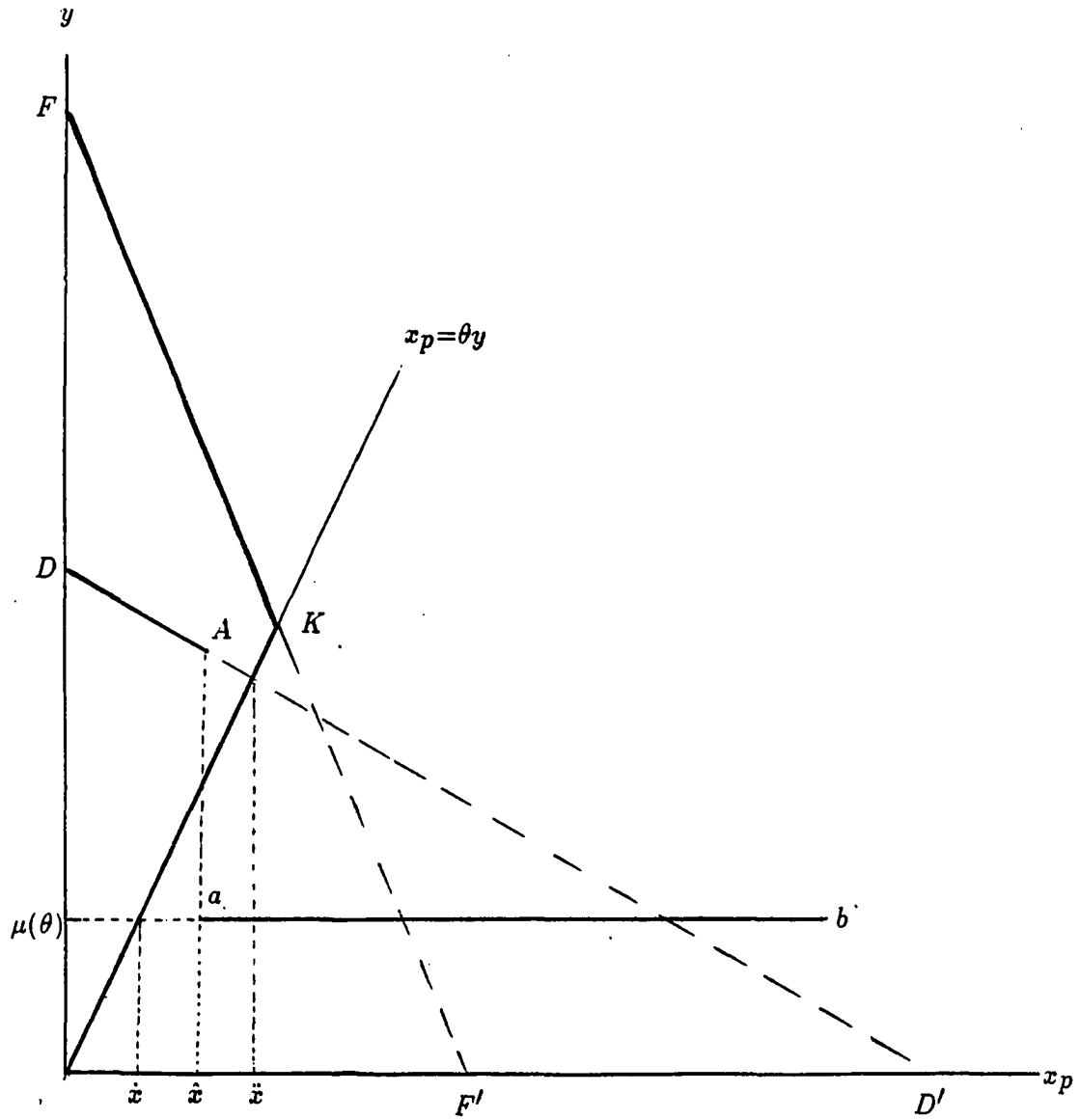


Figure 4.5: Reaction Functions of the Domestic and the Foreign Firm

$$\text{For } x_p \leq \hat{x}(\theta), y^*(x_p, \theta) = B(x_p) > x_p/\theta, \quad (4.12)$$

and

$$\text{For } x_p \geq \hat{x}(\theta), y^*(x_p, \theta) = \mu(\theta) < x_p/\theta, \quad (4.13)$$

with both solutions at  $x_p = \hat{x}(\theta)$ . The home firm's reaction function (for given  $\theta$ ) is represented by the discontinuous locus ( $DAab$ ) in Figure 5.

Let  $\theta^c = x^c/y^c$ , i.e.,  $\theta^c$  is the ratio of imports to domestic production under the unrestricted Cournot solution. By definition,  $y^c = B(x^c)$ . Thus,  $\ddot{x}(\theta^c) = x^c$ , which in turn implies  $\hat{x}(\theta^c) < x^c$ . Since  $\hat{x}$  is increasing in  $\theta$ , there exists  $\theta' > \theta^c$  such that  $\hat{x}(\theta') = x^c$ ; this result is illustrated in Figure 6. In the Figure,  $DD'$  is the domestic firm's reaction function,  $FF'$  the foreign reaction function under free trade, and  $C$  the unrestricted Cournot equilibrium. If a ratio quota equal to  $\theta^c$  is imposed, the home firm's reaction function becomes  $DAGH$  (with a point of discontinuity at  $\hat{x}(\theta^c) < x^c$ ). The value of  $\theta'$  is found by taking the domestic firm's iso-profit curve through the Cournot solution, and finding the ray which is tangent to it ( $OJ$ ); the corresponding domestic reaction function is  $DC'KL$  (with point of discontinuity at  $\hat{x}(\theta') = x^c$ ). Clearly, for  $\theta > \theta'$ , the domestic reaction function includes the Cournot equilibrium point  $C$ .

*Proposition 3:*

For  $\theta < \theta'$ , no pure strategy solution exists with simultaneous play.

*Proof:*

Consider any possible solution,  $x^e$  and  $y^e$ . If  $x^e > \hat{x}(\theta)$ , then  $y^e = \mu(\theta) < x^e/\theta$ . But, from the foreign firm's reaction function, we must have  $x_p \leq \theta y^e$ . Thus, no feasible solution exists for  $x^e > \hat{x}(\theta)$ . Next, suppose  $x^e \leq \hat{x}(\theta)$ ; note that for  $\theta < \theta'$ ,



this implies  $x^e < x^c$ . If  $x^e \leq \hat{x}(\theta)$ , then  $y^e[x^e, \theta] = B(x^e) > x^e/\theta$ , but,  $y^e > x^e/\theta$  also implies  $x^e = \sigma(y^e)$ . Since the unique solution to  $[y = B(x), x = \sigma(y)]$  is the Cournot solution  $(x^c, y^c)$ , there exists no such solution for  $\theta < \theta'$ . *Q.E.D.*

*Corollary:*

If  $\theta \geq \theta'$ , then the unique pure strategy solution is the standard Cournot solution.

*Proof:*

It follows immediately from above. No solution is possible with  $x^e > \hat{x}(\theta)$ , as noted. However,  $x^e \leq \hat{x}(\theta)$  is a feasible solution provided  $x^e = x^c$  (i.e.,  $\theta \geq \theta'$ ). *Q.E.D.*

*Corollary:*

There exist a range of values of  $\theta \in (\theta^c, \theta')$  such that if a ratio quota is imposed at a level which would not be binding under unrestricted trade, this would nevertheless alter the equilibrium.

This latter result means that setting a ratio quota above the unrestricted trade level may still reduce imports (if a solution exists).

#### 4.3.1 Mixed strategy solution

It is well known that a Bertrand–Nash equilibrium does not exist in pure strategies in the presence of capacity constraints.<sup>19</sup> Krishna (1985, 1989) illustrates the nonexistence of a Bertrand–Nash equilibrium through a volume quota, which acts like a capacity constraint on the foreign firm (Krishna, 1989, p. 258). The earlier results of this paper indicates that under a proportional quota, a simultaneous Cournot

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<sup>19</sup>See Fellner (1960, pp. 77–86).



game also entails a mixed strategy solution.

Since, if  $x_p = \hat{x}(\theta)$ , the domestic firm is indifferent between choosing  $y^m = \mu(\theta)$  and  $y^p = B(\hat{x}(\theta))$ , the possibility of a mixed strategy solution exists. Hence:

*Proposition 4:*

Suppose  $\theta < \theta'$ , and that the foreign firm is risk-neutral. Then a mixed strategy<sup>20</sup> solution exists, with the domestic firm randomizing output, while the foreign firm chooses a unique output level (but sales vary). Specifically, the solution is characterized as follows:

With probability  $(1 - \rho)$ ,

$$(x_p, y_p) = (\hat{x}(\theta), \mu(\theta)), \quad (4.14)$$

$$(x, y) = (\theta\mu(\theta), \mu(\theta)), \quad (4.15)$$

where  $\hat{x}(\theta) > \theta\mu(\theta)$ ,

With probability  $\rho$ ,

$$(x_p, y_p) = (\hat{x}(\theta), B(\hat{x}(\theta))), \quad (4.16)$$

$$(x, y) = (\hat{x}(\theta), B(\hat{x}(\theta))), \quad (4.17)$$

where  $B(\hat{x}(\theta)) > \hat{x}(\theta)/\theta$ , and

$$\rho = \frac{\bar{c}'(\hat{x}(\theta))}{p(\hat{x} + B(\hat{x})) + \hat{x}p'} \in (0, 1).$$

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<sup>20</sup>A mixed strategy is defined as probability distribution over all possible pure strategies, where pure strategy is any strategy (or move) done with probability one. The terms, mixed and pure strategy, are widely used by game theorists. See Kreps (1990, Chapters 11-15) and Tirole (1988, Chapter 11) for handy descriptions on mixed strategy.

*Proof:*

If  $x_p \neq \hat{x}(\theta)$ , the domestic firm will not find it optimal to randomize. (there is a unique global profit maximizing solution); we have also shown that no pure strategy solution exists for  $\theta < \theta'$ .

Thus, suppose  $x_p = \hat{x}(\theta)$ ; the domestic firm is indifferent between  $y_1 = B(\hat{x}(\theta))$  and  $y_2 = \mu(\theta)$ , and hence any value of  $\rho$  is equally satisfactory for the home firm.

Given the home firm's strategy,  $y = B(\hat{x})$  with probability  $\rho$  and  $\mu(\theta)$  with probability  $(1 - \rho)$ , the foreign firm chooses  $x_p$  to maximize its expected profits ( $E\bar{\pi}$ ) such that:

$$E\bar{\pi} = \rho[x_1 p(x_1 + B(\hat{x}))] + (1 - \rho)[x_2 p(x_2 + \mu(\theta))] - \bar{c}(x_p), \quad (4.18)$$

subject to  $x_i = \min(x_p, \theta y_i)$ , where  $i = 1$  and  $2$ .

The first order condition, evaluated at  $x_p = \hat{x}(\theta)$ , becomes:

$$\begin{aligned} \frac{\partial E\bar{\pi}}{\partial x_p} &= \rho[p + \hat{x}p'] \frac{\partial x_1}{\partial x_p} + (1 - \rho)[p + x_2 p'] \frac{\partial x_2}{\partial x_p} - \bar{c}'(\hat{x}) \\ &= \rho[p(x_1 + B(\hat{x})) + \hat{x}p' - \bar{c}'(\hat{x})] - (1 - \rho)\bar{c}'(\hat{x}) \\ &= 0, \end{aligned}$$

where, at  $x_p = \hat{x}$ ,  $y_1 = B(\hat{x})$ , so  $x_1 < \theta y_1$ , and  $\frac{\partial x_1}{\partial x_p} = 1$ , while  $y_2 = \mu(\theta)$ , so  $x_p > \theta \mu(\theta)$ , and  $\frac{\partial x_2}{\partial x_p} = 0$ .

Since  $p + \hat{x}p' - \bar{c}'(\hat{x}) > 0$  for  $x_p = \hat{x}(\theta) < x^c$ , and  $y = B(\hat{x}(\theta))$ ,

$$\rho = \left[ \frac{\bar{c}'(\hat{x})}{p(\hat{x} + B(\hat{x})) + \hat{x}p'} \right] \in (0, 1) \quad (4.19)$$

supports the solution. *Q.E.D.*

Note that, as  $\theta$  goes to  $\theta'$ ,  $\hat{x}$  approaches  $x^C$  and  $\rho$  approaches 1.

Given  $\theta < \theta'$ , the domestic firm's profits with the mixed strategy are higher than those at the Cournot equilibrium. When  $\theta$  is zero, the domestic firm enjoys the monopoly profits. As the ratio increases, profits fall to the level of Cournot equilibrium. If the ratio exceeds  $\theta'$ , the domestic firm's profits just stay at Cournot equilibrium. Consumer surplus, however, falls with the mixed strategy because the price rises as the ratio falls. Thus, welfare of the home country becomes ambiguous with the mixed strategy. It is obvious that the MH solution is inferior, for domestic welfare, to simultaneous play for all  $\theta$  because the price is higher at the MH solution than at the simultaneous solution, while the profits with MH are the same as those with the simultaneous solution.

The output of the foreign firm rises as the ratio quota increases.<sup>21</sup> However, the foreign firm's profits with the mixed strategy solution, as compared to the Cournot solution, are ambiguous when  $\theta < \theta'$ . This is because the optimized expected profit function ( $E\bar{\pi}^*(\theta)$ ) of the foreign firm is so highly non-linear that this function is in general not monotone in  $\theta$ . Thus, we could not compare the levels of foreign firm's profits associated with two different ratios. That is, the derivative of the optimized expected profit function of the foreign firm with respect to  $\theta$  is in general indeterminate. Let;

$$E\bar{\pi}^*(\theta) = \rho(\theta)R^1(\theta) + (1 - \rho(\theta))R^2(\theta) - \bar{c}(\hat{x}(\theta)), \quad (4.20)$$

---

<sup>21</sup>Thus, the foreign firm's output with the mixed strategy is smaller than that at Cournot equilibrium.

where  $\rho(\theta)$  is from the equation (4.19) in this chapter,  $R^1(\theta) = \hat{x}(\theta)p[\hat{x}(\theta) + B(\hat{x}(\theta))]$ , and  $R^2(\theta) = \theta\mu(\theta)p[(1 + \theta)\mu(\theta)]$ .

Employing the envelope theorem ( $\rho[p + \hat{x}(\theta)p'] - \bar{c}' = 0$ ), the derivative yields:

$$\begin{aligned} \frac{dE\bar{\pi}^*(\theta)}{d\theta} &= \rho(\theta)\hat{x}(\theta)p'B'\frac{d\hat{x}}{d\theta} \\ &+ (1 - \rho(\theta))\frac{dR^2}{d\theta} + \rho'(\theta)(R^1 - R^2) \end{aligned} \quad (4.21)$$

The first term is positive since  $p'$  and  $B'$  are negative while  $d\hat{x}/d\theta > 0$ . The second term is positive because  $\rho < 1$ , and  $dR^2/d\theta = \theta\mu p'\mu' + \theta(p + \theta\mu p')(\mu' + \mu/\theta) > 0$ , where  $p' < 0$ ,  $\mu' < 0$ ,  $p + \theta\mu p' > p + (1 + \theta)\mu p'$ , which is marginal revenue and always assumed positive, and  $\mu' + \mu/\theta > 0$ .<sup>22</sup> The third term is indeterminate; with an assumption of convex cost function ( $c'' \geq 0$ ),  $\rho'$  is positive, but  $R^1$  may or may not be greater than  $R^2$ . Even if it is striking, it is possible that  $R^2$ , the revenue earned by selling smaller output ( $x_2$  or  $\theta\mu(\theta)$ ) at the higher price ( $p[(1 + \theta)\mu(\theta)]$ ), may be larger than  $R^1$ , the revenue collected by selling larger output ( $\hat{x}(\theta)$ ) at the lower price [ $p(\hat{x} + B(\hat{x}))$ ]. In fact, as we show,  $R^1 < R^2$  in case of linear demand and constant marginal cost.

We employ the same linear demand and constant marginal cost functions as in Section 2;  $p = A - B(x + y)$ ;  $c(y) = cy$ ; and  $\bar{c}(x) = \bar{c}x$ .

Then, using the solution concepts defined in this section, we obtain;  $\hat{x}(\theta) = \left(\frac{A-c}{B}\right)\left(1 - \frac{1}{\sqrt{1+\theta}}\right)$ , and  $y_1(\theta) = B(\hat{x}(\theta)) = \left(\frac{A-c}{2B}\right)\left(\frac{1}{\sqrt{1+\theta}}\right)$   
 $\theta'$  satisfying  $\hat{x}(\theta) = x^c$  becomes:  $\sqrt{1 + \theta'} = \frac{3(A-c)}{2(A-2c+\bar{c})}$ .<sup>23</sup>

<sup>22</sup>Since  $\mu'(\theta)$  is derived from  $\pi_y^i = 0$ ,  $\mu' = -\pi_y^i/\pi_{yy}^i = -\frac{\mu X}{\theta(X - (X - c'')/\theta)}$ .  
 $X = 2p' + (1 + \theta)yp'' < 0$ .  $\mu' + \mu/\theta > 0$ , since  $\frac{X}{X - (X - c'')/\theta} < 1$  if  $c'' \geq 0$ .

<sup>23</sup> $x^c$  was obtained in Section 2.

$y_2 = \mu(\theta) \left( \frac{A-c}{2B} \right) \left( \frac{1}{1+\theta} \right)$ , and  $x_2 = \theta\mu(\theta)$ .

$\rho(\theta) = 2\bar{c}/[-2(A-2c) + 3(A-c)\sqrt{1+\theta}^{-1}]$ ,<sup>24</sup> and  $\rho'|_{\theta=\theta'} = \frac{2(A-2c+\bar{c})^3}{9\bar{c}(A-c)^2}$

$R^1 - R^2 = - \left( \frac{(A-c)(A-3c)}{4B} \right) \left( \frac{(\sqrt{1+\theta}-1)^2}{1+\theta} \right) \leq 0$  if  $A \geq 3c$

Thus;

$$\frac{dE\bar{\pi}^*}{d\theta}|_{\theta=\theta'} = -Z[(A+c)(A-3c) - 6\bar{c}(A - \frac{5}{3}c)] < 0, \quad (4.22)$$

where  $Z = \frac{(A+c-2\bar{c})(A-2c+\bar{c})^3}{162B\bar{c}(A-c)^3} > 0$ , and the square bracket is in general positive assuming that  $A$  is usually far larger than  $c$  and  $\bar{c}$  such that  $A \geq 3c$ , and  $A \geq 6\bar{c}$ .

Thus, the foreign firm's profit function is locally decreasing at  $\theta = \theta'$ , but  $\theta$  near  $\theta'$ . This implies that for  $\theta < \theta'$ , the foreign firm's profits with the mixed strategy are higher than those at Cournot equilibrium in this special case.

In sum, with the mixed strategy, the home firm's profits rise, and consumer surplus falls, so that national welfare is ambiguous, while the foreign firm's output falls, and profits of the foreign firm rise in a special case.

We now turn to the comparison of tariffs and ratio quotas.

#### 4.4 Comparison of Tariff and Quota

In this section, we compare the foreign firm's profits at an excise tariff with those at a ratio quota, generating the same level of domestic firm's profits under both systems. When an excise tariff at a rate of  $t$  is imposed, the profit functions of the two firms become:

$$\bar{\pi}^t(x, y, t) = \bar{\pi}(x, y) - tx, \quad (4.23)$$

$$\pi(x, y) = yp(x+y) - c(y), \quad (4.24)$$

---

<sup>24</sup> $\rho$  equals 1 at  $\theta = \theta'$ .

where  $\bar{\pi} = xp(x + y) = \bar{c}(x)$  as before.

The usual comparative static analysis shows that only the reaction function of the foreign firm shifts to the left, while that of the home firm stays the same. Due to the tariff, the foreign output falls ( $\frac{dx}{dt} < 0$ ) and the home firm's rises ( $\frac{dy}{dt} > 0$ ), with a decline in the total output in the market ( $\frac{d(x+y)}{dt} < 0$ ).<sup>25</sup> Price rises ( $dp/dt = p'(\frac{d(x+y)}{dt}) > 0$ ). Under this tariff system, domestic firm's profits ( $\pi^t$ ) rise ( $\pi_t^t \equiv \frac{d\pi^t(x(t), y(t))}{dt} > 0$ ), while foreign firm's profits ( $\bar{\pi}^t$ ) fall (locally) ( $\bar{\pi}_t^t \equiv \frac{d\bar{\pi}^t(x(t), y(t), t)}{dt} \Big|_{t=0} < 0$ ).<sup>26</sup>

The tariff supporting this equilibrium is obtained from the FOC of the foreign firm such that:

$$t = p(x + y) + xp' - \bar{c}' \quad (4.25)$$

Since this equation is also met along the domestic firm's reaction function, ( $\hat{x}^t$ ,  $B(\hat{x}^t)$ ), a pair of ( $x$ ,  $y$ ) on that reaction function, satisfies:  $p(\hat{x}^t + B(\hat{x}^t)) + \hat{x}^t p' = \hat{t} + \bar{c}'(\hat{x}^t)$ . Then, a ratio quota yielding the same level of home firm's profits at the tariff is represented as follows: We substitute  $\hat{t} + \bar{c}'(\hat{x}^t)$  into  $\frac{\partial E\bar{\pi}}{\partial x_p} \Big|_{x_p=\hat{x}^t} = 0$  to yield  $\hat{t}$  such that:

$$\hat{t}(\theta) = \bar{c}'(\hat{x}^t(\theta)) \left( \frac{1 - \rho(\theta)}{\rho(\theta)} \right) \quad (4.26)$$

Suppose that the home government imposes a ratio quota,  $\theta^t$ , to achieve the

<sup>25</sup>For this system to be stable and unique, we assume that its own second partial in each profit function is more negative than its cross partial, which is a usual assumption.

<sup>26</sup>The home firm's iso-profit curve marches horizontally back to the left, so that it is clear that the tariff increases the home firm's profits as in Figure 7. Since the foreign firm's reaction function shifts due to the tariff, we don't know for sure whether foreign firm's profits rise or fall graphically. A small increase in the tariff reduces the foreign firm's profits as shown in the algebra.

same profits for the home firms as those at  $\hat{t}(\theta)$ . Then, under a ratio quota ( $\theta^t$ ) the domestic firm randomizes its production,  $y_1^t = B(\hat{x}^t)$  and  $y_2^t = \mu(\theta^t)$ , if the foreign firm produces  $\hat{x}^t$ . Thus, the comparison of the foreign firm's profits under the tariff system with those under a ratio quota is similar to the comparison of foreign firm's profits at unrestricted Cournot equilibrium with foreign firm's profits with the mixed strategy. To see the first point, let the profit function of the foreign firm under the tariff ( $\bar{\pi}^t(\hat{t}(\theta))$ ) is:

$$\bar{\pi}^t(t(\theta)) = \bar{\pi}^{t1} - t(\theta)\hat{x}^t, \quad (4.27)$$

where  $\bar{\pi}^{t1}$  is the foreign firm's profits associated with the output under the tariff. Thus,  $\bar{\pi}^t$  is the foreign firm's profit with pure strategy.

Let the profit function of the foreign firm ( $\bar{\pi}^{ms}(\theta)$ ) under a ratio quota is:

$$\bar{\pi}^{ms}(\theta) = \rho\bar{\pi}^1 + (1 - \rho)\bar{\pi}^2, \quad (4.28)$$

where  $\bar{\pi}^1$  is equal to  $\bar{\pi}^{t1}$  at the point  $A$  in Figure 7, and  $\bar{\pi}^2$  is the level of the foreign firm's profits associated with the home firm's lower output,  $y_2(\theta^t)$ . In Figure 7,  $\pi^t$  denotes the profits of the domestic firm under the tariff system at  $A$ .  $\pi^t > \pi^c$ , where  $\pi^c$  is the profits of the home firm at unrestricted Cournot equilibrium.

The comparison of the foreign firm's profits under two different systems is seen by the difference between  $\bar{\pi}^{ms} - \bar{\pi}^t = D$ . Then,  $D$  boils down to:

$$D = \rho\bar{\pi}^1 + (1 - \rho)\bar{\pi}^2 - (\bar{\pi}^{t1} - \hat{t}(\theta)\hat{x}^t) \quad (4.29)$$

$$= (1 - \rho)(\bar{\pi}^2 - \bar{\pi}^1) + \hat{t}(\theta)\hat{x}^t, \quad (4.30)$$

where  $\bar{\pi}^2 - \bar{\pi}^1 = R^2 - R^1$  because the difference between the profits is same as the difference between revenues at the same output of  $\hat{x}^t$ .

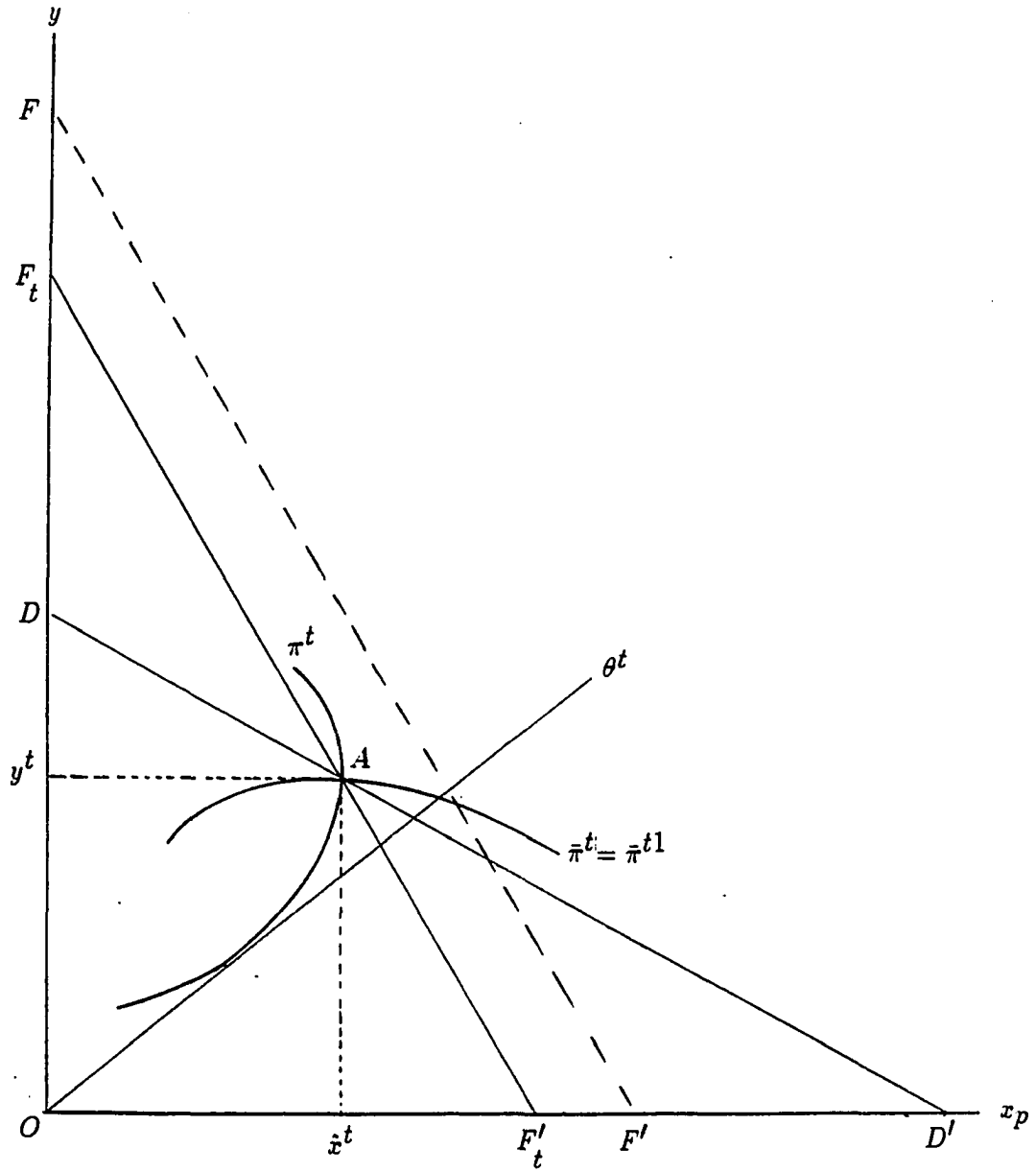


Figure 4.7: Comparison of a Tariff and a Quota



As in the previous section, the sign of  $R^2 - R^1$  is in general indeterminate, but it is positive in a special case of linear demand and constant marginal cost. Thus, in this special case,  $D > 0$ , which implies that the foreign firm prefers a ratio VER to the tariff system yielding the same profit level as a ratio quota to the home firm. Furthermore, the foreign firm prefers a ratio quota to the tariff system even if the home country returns the tariff revenue collected to the foreign country in a lump sum fashion in this special case.

#### 4.5 Concluding Remarks

In this chapter, we point out that the impact of a ratio quota on the equilibrium trade levels depends not only on the magnitude of the quota, but also on the solution concept used to derive the equilibrium. Assuming that the implementation of the quota confers a first play advantage on the home firm, we find the following results; 1) the home firm's maximum profits must be at least as large as the normal Stackelberg profit level; 2) there exists a critical value of the ratio quota ( $\hat{\theta}$ ) such that if a ratio quota is greater than this critical value, the unrestricted Stackelberg solution dominates; 3) if the ratio quota is less than this critical value, the quota will bind; and 4) *a priori*, the critical value may be greater or less than the Cournot free trade ratio. However, the assumption that the presence of the quota changes the game structure, made by MH, is generally unwarranted. That is, the implementation of a ratio quota need not convey a first play advantage on the domestic firm. Thus, we investigate simultaneous play between two firms.

The results under simultaneous play are as follows; 1) no pure strategy solution exists with simultaneous play if the ratio quota is less than the ratio ( $\theta'$ ), at which the

domestic profits are generated at Cournot equilibrium; 2) the unique pure strategy solution is the standard Cournot solution if the ratio quota is greater than or equal to  $\theta'$ ; 3) a ratio quota binds even though a ratio is set above the unrestricted Cournot free trade level; 4) a unique mixed strategy solution exists, with the domestic firm randomizing output, the high with probability  $\rho$ , and the low with probability  $1 - \rho$ , while the foreign firm chooses a unique output level; 5) with the mixed strategy, the home firm's profits rise, and consumer surplus falls, to yield ambiguous domestic welfare; 6) the foreign firm's profits are ambiguous in general, but we show that in a special case of linear demand and constant marginal cost, the profits of the foreign firm with the mixed strategy are **locally** higher.

In comparison of a tariff and a ratio quota, the foreign firm prefers VER to the tariff system in a special case, even if the tariff revenue collected by the home country is returned to the foreign firm by a lump sum.

## 5. SUMMARY AND CONCLUSION

We have shown, in Chapter 2, that the S-B conclusion that optimal intervention by the home government entails cost minimization is incorrect: For given the domestic firm's output, domestic investment affects foreign investment, and hence foreign output, only insofar as it affects the slope of the domestic reaction function in output space. For example, if the slope of the home firm's marginal cost is increasing in the level of its capital, then increased domestic investment reduces the responsiveness of domestic output to foreign output and hence reduces the foreign firm's incentive to overinvest. Thus, it will be optimal for the domestic firm to overinvest; converse results hold if the slope of the home firm's marginal cost is decreasing in the level of capital. Only if the slope does not change as the capital investment changes, it is optimal to minimize costs.

In Chapter 2, we have also shown that the S-B solution is inferior to one in which the home government can, through appropriate policy instruments, precommit both output and investment. This is because for the home country, a Stackelberg leader position gives higher welfare than Cournot.

We have shown, in Chapter 3, that neither of these solutions is time consistent, and have argued that, as long as governments are not permanently empowered, the time consistent solution is more likely to prevail. Surprisingly, we have shown that the

time consistent solution may be superior to either precommitment solution, a result that is at odds with the conventional wisdom concerning the inferiority of the time consistent solution. This result emerges because neither solution is made optimally contingent on foreign investment (which would, of necessity, be a time inconsistent solution).

In Chapter 4, we point out that the impact of a ratio quota on the equilibrium trade levels depends not only on the magnitude of the quota, but also on the solution concept used to derive the equilibrium. Assuming that the implementation of the quota confers a first play advantage on the home firm, we find the following results; 1) the home firm's maximum profits must be at least as large as the normal Stackelberg profit level; 2) there exists a critical value of the ratio quota ( $\hat{\theta}$ ) such that if a ratio is greater than the critical value, the unrestricted Stackelberg solution dominates; 3) if a ratio is less than the critical value, the quota will bind; and 4) *a priori*, the critical value may be greater or less than the Cournot free trade ratio. However, the assumption that the presence of the quota changes the game structure, made by MH, is generally unwarranted. That is, the implementation of a ratio quota need not convey a first play advantage on the domestic firm. Thus, we investigate simultaneous play between two firms.

The results under simultaneous play are as follows; 1) no pure strategy solution exists with simultaneous play if a ratio is less than the ratio ( $\theta'$ ) generating the domestic profits at Cournot equilibrium; 2) the unique pure strategy solution is the standard Cournot solution if a ratio is greater than or equal to  $\theta'$ ; 3) a ratio quota may bind even though it is set above the unrestricted Cournot free trade level; 4) the unique mixed strategy solution exists, with the domestic firm randomizing output,

choosing the high level of output with probability  $\rho$ , and the low with probability  $1 - \rho$ , while the foreign firm chooses a unique output level; 5) with the mixed strategy, the home firm's profits rise, and consumer surplus falls, to yield ambiguous domestic welfare; 6) the foreign firm's profits are ambiguous in general, but we show that in a special case of linear demand and constant marginal cost, profits of the foreign firm with the mixed strategy are **locally** higher.

In comparing the tariff and a ratio quota, the foreign firm prefers the VER to the tariff system, even if the tariff revenue collected by the home country is returned to the foreign firm by a lump sum.

While we have investigated models of strategic trade policy in this dissertation, it is important to recognize that this theory—insofar as it makes policy recommendations—has drawn many criticisms. First, the new theory assumes that the active government has access to the same (complete) information set as the firms in the industry under the examination and that it acts in a way that reflects the national interest, both of which are highly questionable assumptions.

Secondly, the theory has empirical difficulties in formulating the effective strategic policies. For the strategic policy to be effective, we should know detailed information on cost and demand structures of the foreign firms as well as, of home firms. We also should have reliable models of how oligopolists behave; they are cooperative or noncooperative; they are price-setter or quantity-setter. Furthermore, the empirical studies become more difficult when firms in question play a multistage game whose rules and objectives are complex and obscure.

Thirdly, the theory inherently does not consider a general equilibrium aspect. To promote particular sectors means that a government must draw resources away from

the other sectors. A would-be loss from the other sectors which are not supported may dominate the gain from the targeted industry.

Fourth, the strategic trade policy ignores retaliation by the foreign government. Once retaliation is induced, free trade emerges.

Lastly, and most importantly, the new trade theory ignores the political economy of domestic politics. The special interest groups are omnipresent (and to a degree, omnipotent). The strategic trade policy may raise welfare of small, fortunate groups by large amount at costs of larger, more diffuse groups.

This fact, rather, draws a rule of thumb to be established: Free trade. Thus, Krugman said "free trade is not passé—but it is not what it was."

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## APPENDIX A

The notations in this appendix follow S-B (1983).

Each firm  $i$  produces output  $y^i$  at variable cost  $C^i = C^i(y^i, x^i)$ ,  $i = 1$  (home firm) and  $i = 2$  (foreign firm), which includes R & D ( $x^i$ ),<sup>1</sup> and earns revenue  $R^i(y^1, y^2)$ . Since their error stems from the effects of the export subsidy on R & D, we consider only export subsidy ( $z$ ) imposed by the domestic government.

Profit  $\pi^1$  ( $\pi^2$ ) of firm 1 (firm 2) is then:

$$\pi^1 = R^1(y^1, y^2) - C^1(y^1, x^1) + zy^1, \quad (\text{A.1})$$

$$\pi^2 = R^2(y^1, y^2) - C^2(y^2, x^2). \quad (\text{A.2})$$

FOCs are:

$$\pi_1^1 = R_1^1 - c^1 + z = 0, \quad (\text{A.3})$$

$$\pi_2^2 = R_2^2 - c^2 = 0, \quad (\text{A.4})$$

where  $c^i \equiv \partial C^i / \partial y^i > 0$ ,  $i = 1, 2$ .

Solving simultaneously yields:

$$y^1 = q^1(x^1, x^2, z), \quad (\text{A.5})$$

$$y^2 = q^2(x^1, x^2, z). \quad (\text{A.6})$$

---

<sup>1</sup>They assume  $C^i = C^i(y^i, x^i) + v^i x^i$ , where  $v^i$  is a unit cost of R & D. Two cost functions are formally identical.

To see comparative static analysis, we take total differentiation of FOCs to yield:

$$\begin{pmatrix} dq^1 \\ dq^2 \end{pmatrix} = \frac{1}{\Delta} \begin{pmatrix} +\pi_{22}^2 & -\pi_{21}^1 \\ -\pi_{12}^2 & +\pi_{11}^1 \end{pmatrix} \begin{pmatrix} -c_x^1 dx^1 - dz \\ c_x^2 dx^2 \end{pmatrix}, \quad (\text{A.7})$$

where  $\Delta = \pi_{11}^1 \pi_{22}^2 - \pi_{12}^2 \pi_{21}^1$ , and  $\Delta$  is assumed to be positive.

Thus, we have:

$$\partial q^1 / \partial x^1 \equiv q_1^1 = -c_x^1 q_z^1, \quad (\text{A.8})$$

$$\partial q^2 / \partial x^1 \equiv q_1^2 = -c_x^1 q_z^2, \quad (\text{A.9})$$

$$\partial q^1 / \partial x^2 \equiv q_2^1 = -c_x^1 q_z^1, \quad (\text{A.10})$$

where  $q_z^1 = -\pi_{22}^2 / \Delta$ ,  $q_z^2 = \pi_{12}^2 / \Delta$ ,

and  $q_x^1 = -\pi_{21}^1 c_x^2 / \Delta \equiv \gamma(q^1(x^1, x^2, z), q^2(x^1, x^2, z), x^1, x^2)$ . Notice that  $\gamma$  is not only a function of  $q^1$  and  $q^2$ , but  $x^1$  and  $x^2$ , because so are  $\pi_1^1$  and  $\pi_2^2$ , which are in  $\Delta$ .

Then, we have:

$$q_{2z}^1 = \gamma_{y1} q_z^1 + \gamma_{y2} q_z^2, \quad (\text{A.11})$$

$$q_{21}^1 = \gamma_{y1} q_1^1 + \gamma_{y2} q_1^2 + \gamma_{x1}, \quad (\text{A.12})$$

$$= \gamma_{x1} - c_x^1 (\gamma_{y1} q_z^1 + \gamma_{y2} q_z^2). \quad (\text{A.13})$$

At the preceding stage in which firms choose  $x^i$  simultaneously given the rule in the last stage, (net) profit function for the foreign firm is defined as:

$$g^2(x^1, x^2, z) = R^2(q^1, q^2) - C^2(q^2, x^2). \quad (\text{A.14})$$

The FOC, using the envelope theorem, becomes:

$$g_2^2 = -C_x^2 + R_1^2 q_2^1 = 0. \quad (\text{A.15})$$

Since following cross partial is derived:

$$g_{2z}^2 = -c_x^2 q_z^2 + (R_{11}^2 q_z^1 + R_{12}^2 q_z^2) q_2^1 + R_1^2 q_{2z}^1, \quad (\text{A.16})$$

we see that:

$$\begin{aligned} g_{21}^2 &= -c_x^2 q_1^2 + (R_{11}^2 q_1^1 + R_{12}^2 q_1^2) q_2^1 + R_1^2 q_{21}^1, \\ &= -c_x^2 (-c_x^1 q_z^2) - c_x^1 (R_{11}^2 q_z^1 + R_{12}^2 q_z^2) q_2^1 + R_1^2 (\gamma_{x1} - c_x^1 (\gamma_{y1} q_z^1 + \gamma_{y2} q_z^2)), \\ &= -c_x^1 [-c_x^2 q_z^2 + (R_{11}^2 q_z^1 + R_{12}^2 q_z^2) q_2^1 + R_2^1 (\gamma_{y1} q_z^1 + \gamma_{y2} q_z^2)] + R_1^2 \gamma_{x1}, \\ &= -c_x^1 g_{2z}^2 + R_1^2 \gamma_{x1}. \end{aligned} \quad (\text{A.17})$$

(10.B) in page 720 in S-B holds if and only if  $\gamma_{x1} = 0$ .

If  $\gamma_{x1} \neq 0$ , then (10.B) does not hold.